

Asset Pricing with Endogenously Uninsurable Tail Risks

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This paper studies asset pricing implications of idiosyncratic risks in labor productivities in a setting where markets are endogenously incomplete. Well-diversified owners of firms provide insurance to workers using long-term compensation contracts but cannot commit to ventures that yield a negative net present value of dividends. We show that under the optimal contract, workers are uninsured against tail risks in idiosyncratic productivities. Limited commitment makes risk premia higher due to a more volatile stochastic discount factor and a higher exposure of firms' cash-flow to aggregate shocks. Besides salient features of equity and bond markets, the model is consistent with other empirical facts such as the cyclicalities of factor shares and limited stock market participation.

Key words: Equity premium puzzle, dynamic contracting, tail risk, limited commitment

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1 Introduction

This paper presents an incomplete-markets based asset pricing theory that is consistent with observed cross-sectional risk sharing patterns. Recent empirical literature documents the prevalence and systematic movements of idiosyncratic tail risks in labor incomes over the business cycle.¹ Without impediments to risk sharing (as in an Arrow-Debreu world), idiosyncratic fluctuations in income are diversifiable and have no bearing on the market price of aggregate risks. We develop a model that uses limited commitment as a micro-foundation for the absence of insurance markets for tail risks. In contrast to most of the existing literature that assumes exogenously incomplete markets, consumption tail risk in our model is an outcome of optimal dynamic risk sharing arrangements. We show that the lack of risk sharing in our model is both empirically plausible and accounts for higher equity premia along with several other facts such as the cyclicity of factor shares and limited stock market participation.

The setup consists of two types of agents: “capital owners” and “workers”. Capital owners are well-diversified and provide insurance to workers against idiosyncratic fluctuations in labor productivities using long-term compensation contracts. The key friction that distinguishes our paper from standard representative agent asset pricing models is that capital owners cannot commit to contracts that yield a negative net present value of profits from the relationship. We embed this contracting friction in a general equilibrium setting with aggregate shocks and study its asset pricing implications.

Our theoretical framework has several predictions. First, due to limited commitment, tail risks in labor productivities can only be partially insured. In our model, perfect risk sharing implies that all agents in the economy consume a constant share of the aggregate consumption. This Pareto-efficient allocation is implemented by the well-diversified capital owners (the principal) offering a compensation contract to the workers (agents) that is constant across realizations of idiosyncratic productivity shocks. The above arrangement is ex-ante profitable due to the welfare gain of risk sharing. However, a large drop in the level of labor productivity will likely imply that the full risk sharing contract has a negative net present value (NPV) from the principal’s perspective; therefore, extreme productivity shocks cannot be fully insured if the principal cannot commit to ex post negative profit compensation contracts. After calibrating the model to match some key

¹For instance, Guvenen et al. (2014) use a large panel from Social Security Administration to document labor earning patterns across booms and recessions.

features of how labor income dynamics vary over the business cycle, we find that the patterns of earnings losses implied by our optimal contract are in line with the empirical evidence presented in Guvenen et al. (2014).

Taking general equilibrium effects into account, our model generates higher risk premiums through two channels: a more volatile stochastic discount factor and a higher risk exposure of firms' cash flow. First, we find that the capital owners' consumption share is procyclical and more persistent than the underlying aggregate productivity shocks. With recursive utility and persistent countercyclical idiosyncratic risks, the prospect of future lack of risk sharing raises current marginal utilities of workers; the optimal risk sharing scheme compensates by allocating a higher share of aggregate output to the workers. Therefore, labor share moves negatively with the aggregate endowment in our model. The counter-cyclical of labor share translates into a pro-cyclical consumption share of the capital owners and amplifies risk prices. We find that the maximum Sharpe ratio in our model is about three times higher relative to an otherwise identical setting with no agency frictions. These implications of our model are also consistent with Greenwald et al. (2014) who document that variations in factor shares account for a large fraction of stock market fluctuations.

Second, firms' cash flow in our model is more sensitive to macroeconomic shocks than aggregate output. Because under the optimal contract, labor compensation insures workers against aggregate productivity shocks and is counter-cyclical, the residual capital income must be pro-cyclical and more exposed to aggregate shocks. In addition, firms that have experienced adverse productivity shocks are more likely to hit their solvency constraint when aggregate conditions become worse. As a result, they are more exposed to aggregate shocks. In the quantitative analysis, we show that about third of the risk premium for constrained firms comes from their higher cash-flow exposure.

Lastly, the risk sharing arrangement in our model has an interpretation as a theory of endogenous financial market participation. Workers who realize positive favorable productivity shocks are typically unconstrained, and therefore their marginal rate of substitution are equalized with that of the capital owners. However, the limited commitment constraint binds for workers who experience large adverse productivity shocks, and therefore the inter-temporal Euler equation does not hold for such households. This delineation of constrained and unconstrained agents in our optimal risk sharing scheme can be implemented with endogenous market segmentation, where low-income agents do not participate in financial markets for portfolio diversification. In quantitative

exercise, we show that wealthy agents endogenous holder a higher fraction of wealth in the equity market and their consumption is more pro-cyclical, whereas low income workers are constrained and endogenously invest more in riskless assets.

This paper builds on the literature on incomplete market models with limited commitment. Kehoe and Levine (1993) and Alvarez and Jermann (2000) develop a theory of incomplete market based on limited commitment. On the asset pricing side, Alvarez and Jermann (2001) and Chien and Lustig (2009) study the asset pricing implications of binding solvency constraints in models with limited commitment. Most of the above theory build on the Kehoe and Levine (1993) framework and imply that agents who experience large positive income shocks have an incentive to default because they have better outside options. As a result, positive income shocks cannot be insured while tails risks in labor income are perfectly insured. We focus on principal-side limited commitment and our purpose is build a theory of uninsured tail risks and study its asset pricing implications. The principal-side limited commitment problem in our model has a similar structure to those studied in Bolton et al. (2014) and Ai and Li (2015). However, none of these papers allow for aggregate risks and study asset pricing and the equity premium.

Asset pricing models with exogenously incomplete market include Mankiw (1986), Constantinides and Duffie (1996), Krueger and Lustig (2010), Schmidt (2015), and Constantinides and Ghosh (2014). From the theoretical perspective, Krueger and Lustig (2010) provide conditions under which idiosyncratic risks are irrelevant for risk prices, whereas Constantinides and Duffie (1996) present an environment and show that any stochastic discount factor can be constructed by reverse-engineering an appropriate idiosyncratic income process. More recently, Schmidt (2015) and Constantinides and Ghosh (2014) developed quantitative models and demonstrated that the countercyclical labor income risk require a significant compensation. Our paper provides a micro-foundation for the incomplete market exogenously assumed in the above papers, and use empirical evidence on earnings dynamics to discipline the specification of the micro-foundation. In addition, for tractability, models with exogenous incomplete markets typically assume income shocks that are independent over time. In our model, the shocks to workers' earnings is highly persistent under the optimal contract and we show this implication of our model is consistent with the evidence in Guvenen et al. (2014).

Our paper is related to the asset pricing literature on the time variation in idiosyncratic volatility and tail risks. Kelly and Jiang (2014) show that tail risks measured from the cross Section of

equity returns have strong predictive power for aggregate stock market returns. Herskovic et al. (2015) provide evidence that firms' idiosyncratic volatility obeys a strong factor structure and that shocks to idiosyncratic volatility are priced. The above evidences are broadly consistent with our theoretical model.

The theoretical predictions of our model are also consistent with recent literature that emphasize the importance of labor share dynamics in understanding the equity market. Favilukis and Lin (2015) and Favilukis et al. (2016) developed a sticky wage model and demonstrated the importance of counter-cyclical labor share in explaining equity premium and credit risk premium in production economies. Greenwald et al. (2014) and Ludvigson et al. (2014) provided empirical evidence and show that variations in labor share can account for a large fraction of aggregate stock market variations. Lustig et al. (2011) study the dynamics of managerial compensation in a quantitative general equilibrium model with limited commitment.

Our paper is also related to the literature on limited stock market participation and uninsured labor income risks. Danthine and Donaldson (2002) emphasize the importance of operating leverage due to labor compensation. Unlike us, they do not solve for the constrained efficient allocation and endogenize their non-participation result. Berk and Walden (2013) are among early attempts to provide a micro-foundation for limited stock market participation. They focus on heterogeneous endowment but not on agency frictions.

Our computational method builds on the work of Krusell and Smith (1998). Using techniques standard in the dynamic contracting literature such as Thomas and Worrall (1988), Atkeson and Lucas (1992), we represent our equilibrium allocations recursively with the help of distribution of promised values as a state variable. However, in contrast to those papers, our environment has aggregate shocks and the distribution of promised values responds to such shocks even in the ergodic steady state. As in Krusell and Smith (1998), we approximate the forecasting problem of long lived agents by assuming that agents use few relevant moments of the distribution of promised values to guess future state prices.

The paper is organized as follows. We layout the environment - preferences, technology and the key contracting frictions in Section 2. In Section 3 we discuss the optimality conditions and derive the full-commitment benchmark as a point of departure. In Section 4 we derive our key results about endogenous tail risks, procyclical consumption share of the capital owners and implications of operating leverage analytically. Section 5 adds several extensions to the model that help us

confront the model to data and in Section 6 we present the quantitative implications of our model after calibrating it to several U.S. aggregate and cross-sectional facts. Section 7 concludes.

2 The Model

2.1 Model Setup

We consider a discrete time infinite horizon economy with $t = 0, 1, 2, \dots$. There are two groups of agents, a unit measure of capital owners and a unit measure of workers. Preferences are homogeneous across both groups of agents and represented by the Kreps-Porteus form with risk aversion γ and intertemporal elasticity of substitution (IES) ψ . Production takes place in a continuum of firms that each hire one worker using long term wage contracts. The output of firm j at time t , $y_{j,t}$ is determined by

$$y_{j,t} = Y_t s_{j,t},$$

where $s_{j,t}$ is a worker-specific productivity shock and Y_t is the aggregate productivity shock common across all firms. The aggregate technological possibilities evolve stochastically with

$$\frac{Y_{t+1}}{Y_t} = e^{g_{t+1}}, \tag{1}$$

where g_t is a finite state Markov process with transition matrix π and a typical element in π is denote $\pi(g'|g)$. The worker-specific productivity follows

$$\ln s_{j,t+1} - \ln s_{j,t} = \varepsilon_{j,t+1}, \tag{2}$$

with $s_{j,0} = 1$ for all j . Here, $\varepsilon_{j,t}$ is a random shock i.i.d. across firms. We assume $\mathbb{E}[e^\varepsilon] = 1$ so that $s_{j,t}$ can also be interpreted as the firm's productivity relative the economy-wide average productivity. This is a convenient normalization because it implies that $Y_t = \mathbb{E}[y_t]$ is the total output of the economy by the Law of Large Numbers. Importantly, we assume that the distribution of ε depends on the aggregate state of the economy, g and denote the conditional density of ε given g as $f(\varepsilon|g)$. As we show later, this specification allows us to capture the idea that there is more tail risks in labor income and consumption in recessions than booms. We use (g_t, ε_t) to denote time t exogenous shocks and $(g^t, \varepsilon^t) = \{g_s, \varepsilon_s\}_{s=0}^t$ to denote the history of shocks up to time t .

At time 0, workers start with an initial level of productivity y_0 , and enter into a long term contract with the firm that promises them compensations as a function of the histories of idiosyncratic and aggregate shocks that delivers a a life-time utility of U_0 .² We denote a particular compensation contract using $\mathbf{C} = \{C_t(g^t, \varepsilon^t)\}_{t=0}^\infty$.

Capital owners are fully diversified across worker-firm pairs. There is a competitive market between the capital owners for the ownership rights to the firms and a full set of Arrow-Debreu securities. We use $\Lambda_{t,t+j}$ to denote the price of a claim to one unit of consumption in history g^{t+j} denominated in history g^t consumption numeraire, where we suppress its dependence on history to save notation. A firm with contract \mathbf{C} after history g^t is valued as

$$V_t[\mathbf{C}|g^t, \varepsilon^t] = \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} (y_{t+j} - C_{t+j})$$

and a workers utility from \mathbf{C} is calculated according to the Epstein-Zin preference recursion:

$$U_t[\mathbf{C}|g^t, \varepsilon^t] = \left[(1 - \beta) C_t^{1-\frac{1}{\psi}} + \beta \left(\mathbb{E}_t \{ U_{t+1}[\mathbf{C}|g^{t+1}, \varepsilon^{t+1}] \} \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}}$$

We assume that firms with negative net present value can be shut down by the owners and obligations on the compensation contracts are rescinded without any legal recourse. Given the initial condition (\bar{U}_0, y_0) , a firm solves the following problem

$$\begin{aligned} \max_{\mathbf{C}} V_0[\mathbf{C}] \\ U_0[\mathbf{C}] &= \bar{U}_0 \\ V_t[\mathbf{C}|g^t, \varepsilon^t] &\geq 0 \quad \forall (g^t, \varepsilon^t) \end{aligned} \tag{3}$$

Equation (3) is represents firms's lack of commitment and is the only restriction on the set of compensation contracts that can be offered.

With the market structure in place it is easy to see that we can aggregate across the capital owners and denote $X_t(g^t)$ as the consumption of the representative capital owner. To close the model in general equilibrium, we solve for the price system such that the resulting optimal

²Our general framework allows us to draw initial conditions from a joint distribution of y_0 and U_0 as in Atkeson and Lucas (1992). In Section 5 we specify a parsimonious specification that pins down initial utilities of new workers so that the average share of labor compensation is matched to that in the data.

consumption of workers and capital owners satisfy the following market clearing condition after any history g^t :

$$\mathbb{E} [C (g^t, \varepsilon^t) | g^t] + X_t(g^t) = Y_t (g^t).$$

2.2 Recursive formulation

In this section we will characterize the firm's optimal contracting problem recursively and define a notion of a recursive competitive equilibrium for our economy that pins down the equilibrium prices and quantities. We will show that the homogeneity assumptions in preferences and technology imply that the equilibrium we construct will have two state variables, (ϕ, g) , where ϕ summarizes the distribution of agent types and g is the Markov state of aggregate productivity. We use the notation $\Lambda (g' | \phi, g)$ for the one-step-ahead stochastic discount factor (SDF) in state (ϕ, g) . That is, $\Lambda (g' | \phi, g)$ is the price measured in state (ϕ, g) consumption numeraire for one unit of consumption good delivered in the next period contingent on the realization of aggregate shock g' . To make the notation compact, we use $z' = (g', \varepsilon')$ for the vector of realization of next period shocks and $\Omega(dz' | g)$ as the measure over z' given the current aggregate state g . Given our stochastic structure we can factor the joint density $\Omega(dz' | g) \equiv \pi(g' | g) f(d\varepsilon' | g')$, where $f(d\varepsilon' | g')$ is the conditional distribution of ε' in aggregate state g' .

In the recursive formulation, the value of a firm with current output y and a promised utility U to its employee can be calculated as the present value of the worker's total output less that of the compensation promised to the worker:

$$\begin{aligned} V(y, U | \phi, g) &= \max_{C, \{U'(z')\}} \left\{ (y - C) + \int \Lambda (g' | \phi, g) V \left(ye^{g'+\varepsilon'}, U' (z') | \phi', g' \right) \Omega(dz' | g) \right\} \\ U &= \left[(1 - \beta) C^{1-\frac{1}{\psi}} + \beta M (y, U | g)^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}} \\ M (y, U | g) &= \int U' (z')^{1-\gamma} \Omega(dz' | g)^{\frac{1}{1-\gamma}} \\ V \left(ye^{g'+\varepsilon'}, U' (z') | \phi', g' \right) &\geq 0, \text{ for all } z'. \end{aligned}$$

The constraint $V \left(ye^{g'+\varepsilon'}, U' (z') | \phi', g' \right) \geq 0$ is the recursive counterpart of equation (3) in the above optimal contracting problem and reflects the limited commitment on the principal side: capital owners or the “principal” in this contracting relationship cannot commit to contracts the yields negative present value of profit. Due to the homogeneity of the above problem the profit

function satisfies

$$V(y, U | \phi, g) = v\left(\frac{U}{y} | \phi, g\right) y, \quad (4)$$

for some normalized function $v(\cdot | \phi, g)$. We therefore introduce the notation for normalized utility and normalized consumption:

$$u = \frac{U}{y}; \quad c = \frac{C}{y}, \quad (5)$$

and rewrite the above optimal contracting problem as:

$$v(u | \phi, g) = \max_{c, \{u'(z')\}} \left\{ (1-c) + \int \Lambda(g' | \phi, g) e^{g'+\varepsilon'} v(u'(z') | \phi', g') \Omega(dz' | g) \right\} \quad (6)$$

$$s.t: \quad u = \left[(1-\beta) c^{1-\frac{1}{\psi}} + \beta m(u | \phi, g)^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}}$$

$$m(u | g) = \left\{ \int \left[e^{g'+\varepsilon'} u'(z') \right]^{1-\gamma} \Omega(dz' | g) \right\}^{\frac{1}{1-\gamma}} \quad (7)$$

$$v(u'(z') | \phi', g') \geq 0, \text{ for all } z'. \quad (8)$$

2.3 Equilibrium

Let $c(u)$ denote the compensation policy for the optimal contracting problem (6) and let $\Phi(u, y)$ denote the joint distribution of (u, y) . In general, $\Phi(u, y)$ is needed as a state variable in the construction of a recursive equilibrium, because the resource constraint,

$$\int \int c(u) y \Phi(u, y) dy du + X = Y,$$

depends on $\Phi(u, y)$. We can write the integral in the above equation as

$$\begin{aligned} \int \int c(u) y \Phi(u, y) dy du &= Y \int \int c(u) \frac{y}{Y} \Phi(u, y) dy du \\ &= Y \int c(u) \left[\int \frac{y}{Y} \Phi(u, y) dy \right] du. \end{aligned}$$

We define a new measure $\phi(u) = \int \frac{y}{Y} \Phi(u, y) dy$, which will be called summary measure below, and simplify the above resource constraint as

$$\int c(u) \phi(u) du = 1 - x(\phi, g), \quad (9)$$

where $x(\phi, g) = \frac{X}{Y}$ denotes the principal's share in aggregate consumption in state (ϕ, g) . The above procedure reduces the two-dimensional distribution Φ into a one-dimensional measure ϕ and greatly simplifies our numerical computation.

Because we will use ϕ as one of the state variables in the construction of the recursive equilibrium, it is useful to describe the law of motion of ϕ . Let $u' = u'(u, g', \varepsilon' | \phi, g)$ be the law of motion of continuation utility implied by the optimal contracting problem in (6). That is, conditioning on the current state (ϕ, g) , $u' = u'(u, g', \varepsilon' | \phi, g)$ is the continuation utility for an agent with current promised utility u in the next period in the state where (g', ε') are realizations of aggregate and idiosyncratic shocks. Let I be the indicator function, that is, $I_{\{u'(u, g', \varepsilon' | \phi, g) = \tilde{u}\}} = 1$ if and only if $u'(u, g', \varepsilon' | \phi, g) = \tilde{u}$. Under this notation, the law of motion of ϕ is of the form $\phi' = \Gamma(g' | \phi, g)$, where,

$$\forall \tilde{u}, \quad \phi'(\tilde{u}) = \int \phi(u) \int e^{\varepsilon'} f(\varepsilon' | g') I_{\{u'(u, g', \varepsilon' | \phi, g) = \tilde{u}\}} d\varepsilon' du. \quad (10)$$

For simplicity, we specify the equilibrium in terms of normalized consumption and continuation utility as defined in (5). Formally, an equilibrium consists of the following price and allocations:

- A law of motion for ϕ , $\Gamma(g' | \phi, g)$
- A SDF $\{\Lambda(g' | \phi, g)\}_{g'}$
- Capital owners' consumption share $x(\phi, g)$
- Value functions $v(u | \phi, g)$ for each u , and the associated policy functions $c(u | \phi, g)$, $u'(u, g', \varepsilon' | \phi, g)$ such that:

1. The SDF is consistent with the principal's consumption:³

$$\Lambda(g' | \phi, g) = \beta \left[\frac{x(\phi', g') e^{g'}}{x(\phi, g)} \right]^{-\frac{1}{\psi}} \left[\frac{w(\phi', g') e^{g'}}{n(\phi, g)} \right]^{\frac{1}{\psi} - \gamma} \Bigg|_{\phi' = \Gamma(g' | \phi, g)}, \quad (11)$$

where the principal's utility is defined using the recursion

$$w(\phi, g) = \left[(1 - \beta) x(\phi, g)^{1 - \frac{1}{\psi}} + \beta n(\phi, g)^{1 - \frac{1}{\psi}} \right]^{\frac{1}{1 - 1/\psi}}. \quad (12)$$

³Here we specify the SDF as a function of the principal's consumption directly without explicitly specifying the principal's consumption and portfolio problem for brevity. Because the principal is well-diversified, their consumption and investment choices are standard.

and the certainty equivalent, $n(\phi, g)$ in the above equation is defined by

$$n(\phi, g) = \left[\int \left[e^{g'} w(\phi', g') \right] \Omega(dz'|g)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \Big|_{\phi'=\Gamma(g'|\phi, g)}. \quad (13)$$

2. Given the SDF, and the law of motion of ϕ , for each u , the value function and the policy functions solves the optimal contracting problem in (6).
3. Give the policy functions, the law of motion of the summary measure ϕ satisfies (10).
4. The policy functions and the summary measure ϕ satisfy the resource constraint (9).

3 Optimal Contract

To understand the key mechanisms of the model we first derive the optimality conditions that characterize the solution to the contracting problem. As a point of departure, we characterize the equilibrium with full commitment in Section 3.2 and then discuss the implications of limited commitment in Section 4.

3.1 Optimality Conditions

The envelope condition on the optimal contracting problem in equation (6) implies:

$$\frac{\partial}{\partial u} v(u|\phi, g) = - \frac{1}{(1-\beta) \left(\frac{c(u|\phi, g)}{u} \right)^{-\frac{1}{\psi}}}, \quad (14)$$

and the first order condition with respect to continuation utility requires

$$\begin{aligned} & \frac{\beta}{1-\beta} c(u|\phi, g)^{\frac{1}{\psi}} m(u|\phi, g)^{\gamma - \frac{1}{\psi}} \left(e^{g'+\varepsilon'} u'(u, g', \varepsilon') \right)^{-\gamma} \\ & \geq -\Lambda(g'|\phi, g) \frac{\partial}{\partial u} v(u'(u, g', \varepsilon')|\phi', g'), \end{aligned} \quad (15)$$

where we use the notation $m(u|\phi, g)$ for the normalized certainty equivalent of the agent's continuation utility, as defined in (7). Standard arguments show that the firm's value function $v(u|\phi, g)$ is concave and strictly decreasing in u : higher promised utility to the agent implies a lower profit to the principal. Therefore, the limited commitment constraint, $v(u'(u, g', \varepsilon')|\phi', g') \geq 0$ can be written as $u'(u, g', \varepsilon') \leq \bar{u}(\phi', g')$ for all g' , where $\bar{u}(\phi', g')$ is defined as $v(\bar{u}(\phi', g')|\phi', g') = 0$.

The complementary slackness condition implies that (15) has to hold with equality whenever $u'(u, g', \varepsilon') \leq \bar{u}(\phi', g')$.

The above conditions together imply that for all possible realizations of (g', ε') ,

$$\left[\frac{e^{\varepsilon'} c(u'(u, g', \varepsilon' | \phi, g) | \phi', g')}{c(u | \phi, g)} \right]^{-\frac{1}{\psi}} \left[\frac{e^{\varepsilon'} u'(u, g', \varepsilon' | \phi, g)}{m(u | \phi, g)} \right]^{\frac{1}{\psi} - \gamma} \geq \left[\frac{x(\phi', g')}{x(\phi, g)} \right]^{-\frac{1}{\psi}} \left[\frac{w(\phi', g')}{n(\phi, g)} \right]^{\frac{1}{\psi} - \gamma} \quad (16)$$

and $u'(u, g', \varepsilon' | \phi, g) < \bar{u}(\phi', g')$ implies that “=” must hold in (16), where $m(u | \phi, g)$ is defined in (7). If the limited commitment constraint does not bind, then equation (16) is simply the optimal risk-sharing condition that equalizes the marginal utility of the agent with that of the principal. A binding limited commitment constraint allows the marginal utility of the agent exceeds that of the principal, and this is the key mechanism in our model that results in limited risk-sharing with respect to tails risks. Small drops in output y are not associated with reductions in agent’s consumption share relative to that of the principal, because of risk sharing. However, large declines in output send the profit of the insurance contract to the negative region. Because the principal cannot commit to negative NPV insurance contracts, the optimal contract requires a permanent reduction in agent’s future consumption to respect the limited commitment constraint. More negative shocks in output can only be accompanied by further reduction in agent’s consumption — tail risks cannot be fully insured.

3.2 Full Commitment

In this section, we solve the policy functions for an economy with full commitment as a perfect risk sharing benchmark before discussing the model with limited commitment. Because we normalized the utility function to be homogeneous of degree one and because the growth rate of the economy is Markov, the utility of a representative agent who consume the aggregate endowment is linear in Y and can be represented as $u^{FB}(g)Y$ where

$$u^{FB}(g)Y = \left[(1 - \beta)Y^{1 - \frac{1}{\psi}} + \beta \left(\sum_{g'} \pi(g' | g) (u^{FB}(g')Y')^{1 - \gamma} \right)^{\frac{1 - \frac{1}{\psi}}{1 - \gamma}} \right]^{\frac{1}{1 - \frac{1}{\psi}}} \quad \forall g \quad (17)$$

Under recursive preference, the price-to-dividend ratio of the claim to aggregate endowment in this economy, which we denote as $p^{FB}(g)$, can be represented as a function of utility: $p^{FB}(g) = \frac{1}{1-\beta} u^{FB}(g)^{1-\frac{1}{\psi}}$.

With perfect risk sharing, homogeneous preferences imply that the consumption of all agents in the economy must be proportional to each other. Therefore, if the promised utility of an agent is U , he must consume a $\frac{U}{u^{FB}(g)Y}$ fraction of the aggregate consumption stream and the present value of this compensation contract is $\frac{U}{u^{FB}(g)Y} p^{FB}(g) Y = \frac{U}{u^{FB}(g)} p^{FB}(g)$. Using our definition of the normalized value and policy functions, (4) and (5), the normalized value and policy functions in the first best case can be written as:

$$v^{FB}(u) = p^{FB}(g) \left[1 - \frac{u}{u^{FB}(g)} \right], \quad c^{FB}(u) = \frac{u}{u^{FB}(g)}. \quad (18)$$

To derive the policy function for normalized continuation utility $u'(u, g', \varepsilon' | \phi, g)$, consider an agent with normalized promised utility u and current output y . Under perfect risk sharing, he consumes $\frac{uy}{u^{FB}(g)Y}$ fraction of aggregate endowment, and therefore the next period continuation utility is $u'y' = \frac{uy}{u^{FB}(g)Y} u^{FB}(g') Y'$. Therefore,

$$u'(u, g', \varepsilon' | \phi, g) = \frac{u^{FB}(g')}{u^{FB}(g)} u \cdot e^{-\varepsilon'}. \quad (19)$$

Intuitively, because under perfect risk sharing, continuation utility does not respond to idiosyncratic shocks, normalized utility $u'(u, g', \varepsilon' | \phi, g)$ must be inversely proportional to the idiosyncratic shock $e^{\varepsilon'}$.

We make two observations. First, despite risk aversion, perfect risk sharing implies that agents are neutral with respect to idiosyncratic risks. As a result, the marginal cost of utility provision is constant and the consumption policy is a linear function of promised utility, as shown in equation (18). Constant marginal cost of utility provision is the hallmark of perfect risk sharing. In fact, as we show later, under limited commitment, the marginal cost of utility provision will be an increasing function of promised utility u . Second, under the optimal contract with full commitment, equity holders of firms absorb all idiosyncratic risks and firm value can be negative. By equation (19), an extremely negative shock in ε' pushes $u'(u, g', \varepsilon' | \phi, g)$ toward infinity and can result in a negative firm value, $v^{FB}(u)$ in equation (18). Intuitively, perfect risk sharing requires all workers consume the same fraction of aggregate consumption. Firms with worker who are

extremely unproductive ex post must sustain a negative profit. Clearly, the perfect risk sharing contract violates constraint (8) and is therefore not feasible under limited commitment.

Finally, we note that the normalized continuation utility u can be interpreted as a measure of worker's share in the firm's valuation, because U is the total utility delivered by the compensation contract and y is firm size. In what follows, we will use the term normalized continuation utility and workers' share interchangeably.

4 Limited Commitment

In this section we present three analytical results for our model and demonstrate how limited commitment enhances the equity premium. First, under the optimal contract with limited commitment, idiosyncratic tail risks, i.e., a sufficiently adverse realization of ε draws, are not hedged. Second, combined with recursive preferences, persistent countercyclical idiosyncratic risks translate into a countercyclical labor share and a procyclical consumption share of well-diversified capital owners, making the stochastic discount factor more volatile. Third, the optimal contract generates a form of operating leverage, making dividend payment more risky than aggregate consumption. We show that the optimal compensation contract should insure workers against aggregate shocks and as a result, payment to capital owners is more sensitive to aggregate shocks than that in standard representative agent models.

4.1 Lack of insurance against tail risks

We first present a proposition that formalizes the notion that the lack of commitment on the principal side limits the insurance against tail risks provided by compensation contracts. Uninsured tail risks are important for the quantitative implications of our model, because exposure to tail risks strongly affects agent's marginal utility and generates a significant volatility of the stochastic discount factor in equilibrium.

To fix ideas, it is useful to note that under the optimal contract, the elasticity of $u'(u, g', \varepsilon' | \phi, g)$ with respect to ε' must be between -1 and 0 . As we explain in the previous section, under full commitment, the elasticity of $u'(u, \varepsilon', g' | \phi, g)$ with respect to ε' is -1 (see equation (19)). That is, in order for the continuation utility U not to respond to idiosyncratic shocks, normalized utilities must move one-to-one in the opposite direction of the realization of ε' . On the other hand, in the

absence of any risk sharing, the above elasticity must be zero. To see this, consider an economy in which all workers consume their own output. Because the ε shocks are i.i.d. over time, workers' utility must be proportional to y and the elasticity of $u'(u, \varepsilon', g'|\phi, g)$ with respect to ε' must be zero. In our model, agency frictions limits but does not completely eliminate risk sharing, and as a result, we obtain an elasticity between -1 and 0 .

The following proposition shows that when the limited commitment constraint is binding in our model, the optimal contract exhibits an extremely form of lack of risk sharing for productivity shocks in the left tail of its distribution: the elasticity of $u'(u, \varepsilon', g'|\phi, g)$ with respect to ε' is zero. Recall that $\bar{u}(\phi, g)$ is defined as the level of normalized utility at which the limited commitment constraint binds, that is, $p(\bar{u}(\phi, g)|\phi, g) = 0$. Below, we characterize the properties of $u'(u, \varepsilon', g'|\phi, g)$ under the optimal contract with limited commitment.

Proposition 1. *There exists $\underline{\varepsilon}(u, g'|\phi, g)$ with $\frac{\partial \underline{\varepsilon}(u, g'|\phi, g)}{\partial u} > 0$ such that*

$$u'(u, \varepsilon', g'|\phi, g) = \bar{u}(\Gamma(g'|\phi, g), g') \quad \text{for all } \varepsilon' \leq \underline{\varepsilon}(u, g'|\phi, g),$$

and $u'(u, \varepsilon', g'|\phi, g)$ is strictly decreasing in ε' for all $\varepsilon' > \underline{\varepsilon}(u, g'|\phi, g)$.

By the above proposition, at $\varepsilon' = \underline{\varepsilon}(u|g', \phi, g)$, $u' = \bar{u}(g', \Gamma(g'|\phi, g))$, and lower realizations of ε' are not associated with increases in u' . As a result, the unnormalized continuation utility, $U' \propto e^{-\varepsilon'} u'$ falls with ε' and so do the levels of future labor compensation. Clearly, tail risk are not insured under the optimal contract, as continuation utility and earnings fall proportionally with negative shocks for all $\varepsilon' \in (-\infty, \underline{\varepsilon}(u|g', \phi, g))$.

To illustrate the implications of limited commitment, we plot the firm's normalized valuation and the policy rules for worker's normalized compensation and normalized promised values. In Figure 1 we see that the normalized value of the firm as a function of promised value is downward sloping and strictly concave with limited commitment as against linear with full commitment. The higher marginal cost to the firm of insuring a worker with promised values close to \bar{u} comes from the possibility of future tail risks. In Figure 2, we plot the normalized continuation utility as a function of the realization of the idiosyncratic productivity shock ε' for two levels of current period utility, u . We confirm the findings of the above proposition: normalized promised utilities are constant for ε' shocks in the left tail of its distribution. Further, for $u_{high} > u_{low}$, we see the threshold $\underline{\varepsilon}(u_{high}, g'|\phi, g) > \underline{\varepsilon}(u_{low}, g'|\phi, g)$: ceteris paribus, firms with high u are more likely to be

constrained. In the quantitative analysis in Section 6, we will show that the risk sharing patterns that emerge out of these dynamics of continuation values are consistent with several empirical observations about individual earnings risk in data.

4.2 General equilibrium implications

The tail risk property in Proposition 1 is derived using the properties of the optimal contract problem (6) and it holds for any arbitrary stochastic discount factor. However, to understand how uninsured tail risks affect the properties of the equilibrium stochastic discount factor, we need to take a step further and study the implications of market clearing and general equilibrium. In this section, we study an analytical example to demonstrate the mechanism in our model that generates a more volatile stochastic discount factor and a higher risk exposure of firms' cash flow. Our example highlights the importance of the assumptions of preference for early resolution of uncertainty and countercyclical idiosyncratic risks.

At the level of generality that we introduced in Section 2, the model does not have an analytical solution. In this section, we consider a special case by making the following assumptions.

Assumption 1. *Aggregate shocks are absorbing, i.e., $\pi(g'|g) = 1$ if $g' = g$, The idiosyncratic shocks $f(\varepsilon|g = g_H)$ is degenerate and $f(\varepsilon|g = g_L)$ is a negative exponential with parameter λ .⁴*

Assumption 2. *Consumption for $t \geq 2$ is a constant fraction of α of firm revenue, i.e., $C_t = \alpha y_t$.*

Assumption 3. *Preferences satisfy $\gamma \geq \psi = 1$*

Assumptions 1-2 deliver us a setting where uninsurable idiosyncratic risks are countercyclical and persistent. For tractability, we assume a simple form of lack of risk sharing starting from period 2. This assumption will be relaxed in our full model where the lack of risk sharing is endogenously determined by the optimal dynamic contract. The assumption of unit elasticity of intertemporal substitution is merely for tractability and will be relaxed in the quantitative exercise.

Procyclical dividend share Our first result is that fixing IES, a sufficiently high risk aversion is both necessary and sufficient for the consumption share of capital owners to be procyclical. Because capital owners are unconstrained, their marginal rate of substitution must be the

⁴See Appendix A for the description and key properties of the negative exponential distribution.

relevant stochastic discount factor for all assets in the economy. As a result, the procyclicality of capital share makes the stochastic discount factor more volatile in our framework relative to an otherwise identical model with full risk sharing.

Under Assumption 1-3, the only non-trivial period is $t = 1$ where the aggregate state is determined for the rest of the time. In this setup, there is no need to use distribution as a state variable, and we simply denote $x(g_H)$ and $x(g_L)$ to be the consumption share of the well-diversified capital owner at $t = 1$ in state g_H and g_L , respectively.

Proposition 2. *Under Assumptions 1-3 there exists a $\gamma^{max} \in [1, 1 + \lambda)$ such that $\gamma > \gamma^{max}$ implies $x(g_H) > x(g_L)$. Moreover as $\gamma \rightarrow 1$, $x(g_H) < x(g_L)$.*

The cyclicity of capital share depends on the balance of two forces. The first comes from market clearing condition. Note that optimal risk sharing implies that the marginal utility of all unconstrained agents must be equalized. Countercyclical idiosyncratic risk implies that relative to booms, a larger fraction of agents get constrained in recessions. Since constrained firms cut compensation, in the aggregate there are more resources available. Optimal risk sharing requires equating intertemporal rates of substitutions, which amounts to equalizing the growth rates of consumption of the capital owners and the unconstrained agents under expected utility. Therefore, for $\gamma = 1 = \frac{1}{\psi}$, the consumption share of both capital owners and unconstrained agents must increase and $x(g_L) > x(g_H)$.

The second force is activated when we depart from expected utility. As risk aversion exceeds the inverse of intertemporal elasticity of substitution, marginal utilities are decreasing in both current consumption and continuation values. Recessions that are persistent and associated with lack of risk sharing in the future imply lower continuation values and a higher marginal utilities in the current period for currently unconstrained agents. These conditions imply that optimal risk sharing is now achieved by transferring resources away from the capital owners to these unconstrained agents. Proposition 2 says that for sufficiently high risk aversion, this incentive is strong enough to dominate the effect of market clearing and deliver procyclical consumption shares for capital owners.

Operating leverage Our second result is that the countercyclical labor compensation contract creates a form of operating leverage and elevates the risk exposure of dividends relative to that in an otherwise identical representative firm model. Our key result, Proposition 3 below, states that risk exposure is an increasing function of worker's equity share in the firm, u_0 .

We first set up some notation. Consider a firm with worker's equity share u_0 in period zero. The value of the firm in period 1 is $v(u'(u_0, g', \varepsilon') | g') y_0 e^{g' + \varepsilon'}$. The exposure of this firm's equity with respect to aggregate productivity shock is determined by the ratio of firm value across aggregate states, g_H and g_L :

$$\frac{\mathbb{E} \left[v(u'(u_0, g_H, \varepsilon') | g_H) y_0 e^{g_H + \varepsilon'} \middle| g_H \right]}{\mathbb{E} \left[v(u'(u_0, g_L, \varepsilon') | g_L) y_0 e^{g_L + \varepsilon'} \middle| g_L \right]}.$$

To establish that firms' risk exposure is increasing in u_0 , it is enough to show $\frac{d}{du_0} \Delta(u_0) > 0$, where

$$\Delta(u_0) = \frac{\mathbb{E} \left[v(u'(u_0, g_H, \varepsilon') | g_H) e^{\varepsilon'} \middle| g_H \right]}{\mathbb{E} \left[v(u'(u_0, g_L, \varepsilon') | g_L) e^{\varepsilon'} \middle| g_L \right]}. \quad (20)$$

Proposition 3. *Under Assumptions 1-3 we have $\frac{d}{du_0} \Delta(u_0) > 0$ as $\gamma \rightarrow 1 + \lambda$.*

Proposition 3 provides a theory for an endogenously higher cash flow exposure. Note that as u_0 approaches zero, the compensation to workers approaches to zero as well. Therefore, $v(u'(u_0, g, \varepsilon') | g)$ converge to the price-to-dividend ratio of the aggregate endowment. An immediate implication of the above Proposition is that the dividend flow of all firms in the economy have a higher risk exposure than aggregate endowment.

As we increase u_0 , promised payments to workers as a share of firm's revenues increases in both booms and recessions. Under the full commitment benchmark, this increase is proportional. Cash flow of all firms, regardless of their leverage, are equally sensitive to the aggregate shocks. Thus risk premium as a function of u_0 is constant and equal to that of the claim on aggregate endowment. With limited commitment, it is optimal to deliver the promised commitments to workers in states where the marginal utility of worker is lower. As discussed before, with risk aversion sufficiently higher than IES, most of the movement in marginal utility are driven by concerns of future binding constraints and increasingly for firms with higher threshold $\underline{\varepsilon}$. Thus the residual cash flows, i.e., dividends of such firms drop more in recessions relative to in booms. Along with operating leverage that we established in Proposition 2, we see why firms with higher u_0 command higher equity premium.

5 Extensions

In order to confront our model with data we allow for three extensions to the baseline setting. Although not crucial for the key asset pricing implications, these extensions allow our model to capture some key quantitative features of the data, and therefore make it possible for us to use relevant moments in the data to discipline parameter choices in calibration.

First, we modify the the aggregate productivity process to allow for unpredictable shocks to its growth rates with stochastic volatility in order to better match the time series properties of aggregate consumption. We change equation (1) to

$$\frac{Y_{t+1}}{Y_t} = e^{g_{t+1} + \sigma(g_{t+1})\eta_{t+1}}, \quad (21)$$

where g_t is a finite state Markov process as before and η_t is i.i.d standard Gaussian. We allow the volatility of the Gaussian component $\sigma(g)$ to depend on the persistent aggregate state, g . In Section 6, we estimate the parameters in (21) using aggregate consumption data and show that this specification allows our model to match well the key moments of aggregate consumption growth rates.

The above extension introduces an addition complication in the computation of equilibrium, because all equilibrium objects, in principle, could now depend the an extra state variable η . However, we show below that the equilibrium prices and the optimal contract satisfy a homogeneity property, and the presence of η shocks does not increase the state space of the equilibrium value and policy functions. In particular,

Lemma 1. *For all u , all (g', ε') and $\eta'_1 \neq \eta'_2$, $u'(u, \varepsilon', g', \eta'_1 | \phi, g) = u'(u, \varepsilon', g', \eta'_2 | \phi, g)$.*

By the above lemma, the normalized continuation utility $u'(u, \varepsilon', g', \eta' | \phi, g)$ does not depend on the realization of η shock. Intuitively, because the principal and the agent have identical homothetic preferences, optimal risk sharing requires that they both consume a fixed proportion of aggregate output unless doing so affects the incentive compatibility constraint, (8). The realization of η shock is i.i.d. over time and does not affect the conditional distribution of ε' , therefore it is feasible to maintain a constant share after such shocks. It follows that the optimal contract requires so. By the above lemma, our construction of the equilibrium in Section 2.3 remains valid with a slight

modification of the SDF as follows:

$$\Lambda(g', \eta' | \phi, g) = \beta \left[\frac{x(\phi', g') e^{g' + \sigma(\eta')\eta'}}{x(\phi, g)} \right]^{-\frac{1}{\psi}} \left[\frac{w(\phi', g') e^{g' + \sigma(\eta')\eta'}}{n(\phi, g)} \right]^{\frac{1}{\psi} - \gamma} \Bigg|_{\phi' = \Gamma(\phi | g, g', \eta')} . \quad (22)$$

Our second modification is to allow for agent side limited commitment in the same way as in Kehoe and Levine (1993) and Alvarez and Jermann (2000) in order to better match the dynamics of individual earnings in the data. We assume that workers cannot commit to compensation contracts that deliver lower utility than their outside options. Workers outside options, \underline{U}_t , takes the form of $\underline{U}_t \equiv y_t \underline{u}(g_t)$, where $\underline{u}(g_t)$ depends only on the aggregate state of the economy. This additional friction amounts to adding another constraint, $U' \geq \underline{u}(g')y'$ into the optimal contracting problem in (6).

The principal-side limited commitment constraint in equation (6), $V(ye^{g'+\varepsilon'}, U'(z') | \phi', g') \geq 0$, requires workers' promised utility, U_t to go down relative the rest of the economy whenever it binds. However, in the absence of any other frictions, risk sharing requires that continuation utility U_t should never increase. As a result, the model implied earnings process follows a similar pattern, and worker rarely experience extended compensation increases. Adding agent-side limited commitment helps correct the above counterfactual implications of the model. The agent side limited commitment constraint, whenever binding, requires U_t and future compensation to increase to match workers' outside options. As a result, our full model is able to capture the rich dynamics of individuals' earnings process and allows us to calibrate our model to such empirical evidence.

Incorporating limited commitment on agent side in our framework is straightforward. Analogous to the case of limited commitment on principal side, optimal contracting with two-sided limited commitment boils down to adding the following extra constraint to the contracting problem 6:

$$u'(\varepsilon', g') \geq \underline{u}(g') \quad \forall g'. \quad (23)$$

The implication of constraint (23) is that now there exists a $\bar{\varepsilon}(u, g' | \phi, g)$ such that the risk sharing condition (16) holds with equality for $\underline{\varepsilon}(u, g' | \phi, g) \leq \varepsilon' \leq \bar{\varepsilon}(u, g' | \phi, g)$ and $u'(u, \varepsilon', g' | \phi, g) = \underline{u}(g')$ for $\varepsilon' \geq \bar{\varepsilon}(u, g' | \phi, g)$. This means that for a sufficiently high realization of ε' the contract increases labor compensation so that the worker is incentivized to stay in the contract.

The last extension is that we allow entry and exit of workers to maintain a stationary long-

run distribution of the model. For tractability, we have assumed that the individual productivity process to be a geometric random walk, which does not have a long-run stationary distribution. A simple device to correct this to allow existing workers to exit the work force at rate $\kappa > 0$ per period and assume a measure $\frac{1}{\kappa}$ of new workers start every period at $s_0 = 1$. The assumption ensures that the total measure of workers in the economy is always one, and the distribution of idiosyncratic productivity is stationary.

Clearly, our baseline setting of Section 2 is the special case in which $\sigma(g) = \underline{u}(g) = \kappa = 0$ for all g . In the next section, we explain how to use empirical evidence to discipline these parameters in the general setup and how the extended model allows us to match a broader set of moments in the data.

6 Quantitative Analysis

In this section we discuss the numerical implementation and quantitative implications of our model. We first discuss a notion of an “approximate” equilibrium that can be implemented using a Krusell-Smith type algorithm. Later calibrating parameters to match the aggregate and cross sectional moments for U.S. data, we quantify the effect of tail risk on cross-sectional earnings losses and returns across firms that differ in their operating leverage.

6.1 Numerical algorithm and calibration

The key computational challenge in our setup is that the distribution ϕ is an infinite dimensional state variable. We use a numerical procedure on the lines of Krusell and Smith (1998) and replace ϕ with suitable summary statistics. The distribution ϕ enters the problem through its effect on the stochastic discount factor and market clearing. To approximate the law of motion Γ , we conjecture that agents project down the stochastic discount factor on the space spanned by $\{g_t, x_t\}$ and use

$$x' = \Gamma_x(g'|g, x), \tag{24}$$

as the law of motion for the transition of x_t . Replacing ϕ with x and $\Gamma(\cdot)$ with $\Gamma_x(\cdot)$, one can easily compute the SDF $\Lambda(g', \eta'|g, x)$ from equations (22). Through market clearing condition in equation (9) we observe that the x appropriately summarizes higher moments of ϕ as the function

$c(\cdot)$ is strictly increasing and convex. This choice contrasts our algorithm from that in Krusell and Smith (1998) who use the first moment of the distribution of wealth as a sufficient state statistic.⁵

Now given $\Lambda(g', \eta'|g, x)$, we can solve the optimal contract to obtain optimal compensation $c(u|g, x)$ and continuation values $u'(\varepsilon', g', u|g, x)$. The policy functions $\Gamma_x(g'|g, x)$, $u'(\varepsilon', g', u|g, x)$, the probability distribution $f(\varepsilon'|g)$, together with the transition matrix $\pi(g'|g)$ jointly define a Markov process (x_t, u_t, g_t) . Denote the ergodic distribution of this Markov process as $\Psi(u, x, g)$. The *approximate* equilibrium is defined by the fixed point of the following functional equation in Γ_x :

$$\Gamma_x(g'|g, x) = 1 - \int \int e^{\varepsilon'} c(u'(u, \varepsilon', g'|g, x)|g', \Gamma_x(g'|g, x)) f(\varepsilon'|g) \Psi(du|x, g) \quad (25)$$

Appendix B describes the detailed steps and diagnostics necessary to implement this fixed point numerically.

Calibration

In Section 4 we discussed that our setup generates endogenous tail risk in labor earnings and along with recursive preference this tail risk can manifest in a volatile stochastic discount factor through the operating leverage channel. In this section we use the numerical method sketched out in the previous section to assess the quantitative relevance of this mechanism. The task at hand is to take a stand on the underlying distribution of idiosyncratic productivities and how it varies over the business cycle. For this purpose we will use moments from Guvenen et al. (2014) that uses detailed administrative data from Social Security Administration to document the cyclical properties of earnings over business cycle for the sample 1979-2010 and calibrate $f(\varepsilon|g)$ such that key moments of the endogenous distribution of labor compensation in our model match those of the earnings distribution in data. We will then analyze the model's asset pricing properties.

We follow the standard practice of calibrating our model at the quarterly level and time aggregate outcomes in our model because most of the moments we match in the data are available

⁵Using x as the state variable is both numerically efficiently and computationally convenient. Note that the stochastic discount factor depends on ϕ for two reasons. First, ϕ affects capital owners consumption, and this effect is completely summarized by x . Second, ϕ is a sufficient statistic to forecast future prices. We show that the R^2 of a linearized version of (24) is as high as 99.9% in our numerical solution. Our method is numerically efficient because given the law of motion of x , the equilibrium stochastic discount factor is completely determined without solving for the optimal contract.

at the annual level. We calibrate the parameters of aggregate productivity process using aggregate consumption data. We assume g is a two-state Markov chain with state space $\{g_H, g_L\}$ and refer to “booms” as states with $g = g_H$ and “recessions” as states with $g = g_L$. The aggregate shock process $\{g, \eta\}_t$ are calibrated as in Ai and Kiku (2013) who jointly estimate the levels of g_H, g_L , the Markov transition matrix, and the volatility parameters, $\sigma(g_H)$ and $\sigma(g_L)$ from post-war aggregate consumption data. Our calibration imply an average duration of 12 years for booms and 4 years for recessions. The parameters for the aggregate shocks are summarized in Table 1.

We calibrate the worker specific parameters, that is, κ , $\underline{u}(g)$, $u^*(g)$, and parameters for the distribution $f(\varepsilon|g)$, to match the micro level evidence on earnings dynamics as documented in Guvenen et al. (2014). To maintain parsimony we assume that $f(\varepsilon|g)$ is Gaussian in booms and follows a mixture of a Gaussian and negative exponential in recessions. The negative exponential distribution in recessions allows us to capture the idea of elevated tail risks in recessions, and allows the model to match the difference in the skewness of earnings distribution across booms and recessions as in Guvenen et al. (2014). This assumption leaves us with two parameters $\{\mu_H, \sigma_H\}$ for booms and $\{\mu_L, \sigma_L\}$ for the Gaussian distribution in recessions, $\{\lambda_L, \varepsilon_L^{max}\}$ for the negative exponential and $\rho_L \in (0, 1)$ as the mixing probability that represents the probability of a draw of the negative exponential. Since shocks ε are modeled as shares we have $\int e^\varepsilon f(\varepsilon|g)d\varepsilon = 1$. We further assume that the conditional mean is 1 for each of the individual distribution in the mixture too. These restrictions imply

$$\mu_H = -\frac{\sigma_H^2}{2}, \quad \mu_L = -\frac{\sigma_L^2}{2}, \quad \text{and } \varepsilon_L^{max} = \log \frac{1 + \lambda_L}{\lambda_L},$$

and we are left with five parameters $\{\sigma_H, \mu_L, \lambda_L, \rho_L\}$ that completely pin down $f(\varepsilon|g)$.

For the rest of the parameters we set $\underline{u}(g) = \underline{\alpha}u^{FB}(g)$ where $u^{FB}(g)$ are defined in (17) and correspond to the normalized utilities in the full commitment case. We set $\kappa = 1\%$ per quarter to reflect an average duration of staying in the work force for 25 years. We parameterize $u^*(g) = \bar{\alpha}u^{FB}(g)$. In the data, we first average the cross-sectional moments of one year earnings growth across booms and recessions using the same definitions recessions as Guvenen et al. (2014). We choose 6 parameters $\{\sigma_H, \sigma_L, \lambda_L, \rho_L, \underline{\alpha}, \bar{\alpha}\}$ to match the moments reported in Table 2. The table also reports the fit.

All the parameters reported are jointly calibrated and in general there is no exact one-to-one

map between the moments targeted and individual parameters. However by varying one parameter at a time we can uncover which parameters are informative about which moments in an approximate way. The choice of σ_H and σ_L pin down the cross-sectional standard deviation of earnings growth which in data is similar across boom and recessions. We estimate a λ_L and ρ_L which implies a small probability of a draw from a significantly left-skewed distribution. This mainly accounts for the excess skewness in recessions relative to booms. An important observation is that the possibility of the tail risk and not its recurrent occurrence along a path is enough for the operating leverage to be at work and the realized skewness is driven by implications of the optimal contract. As is well known in several settings with incomplete markets, over extremely low frequencies, the optimal contract drifts towards region of the state space where agency frictions are minimized. In the context of our model this implies that the aggregate share of labor compensation approaches high value. Parameters $\{\underline{\alpha}, \bar{\alpha}\}$ shift the earnings distribution and are helpful in increasing the labor share. For example when we set $\underline{\alpha}$ to zero, the labor share approaches 15%. We found that our results are not too sensitive to κ as long it is low and in the range of 1% to 3%.

6.2 Results

Using parameters in Table 1 we apply our adaptation of the Krussel-Smith procedure to compute the optimal contract and law of motion for x_t . We use a log-linear specification for the law of motion of x , (24) with flexible coefficients:

$$\log x_{t+1} = \sum_{g,g'} \Gamma_0(g, g') I_{\{g_t=g \quad g_{t+1}=g'\}} + \sum_{g,g'} \Gamma_1(g, g') I_{\{g_t=g \quad g_{t+1}=g'\}} \log x_t, \quad (26)$$

where $I_{\{g_t=g \quad g_{t+1}=g'\}} \neq 0$ only if $g_t = g$ and $g_{t+1} = g'$. That is, we allow the regression coefficients in (26) to depend on the aggregate states. In simulations, the above law of motion summarizes the equilibrium dynamics of x very well, the R^2 of the above regression is above 99.9% after convergence.

Before discussing the asset pricing moments, we first show that our calibrated model captures some key features of the the dynamics of individual earnings reported in recent empirical works, for example, Guvenen et al. (2014). This aspect of our framework compares favorably to asset pricing models with exogenously incomplete markets, for example, Constantinides and Duffie (1996), Schmidt (2015), and Constantinides and Ghosh (2014), that in order to insure no trade in equilibrium, typically assume quite stylized and counter factual dynamics for workers' earnings

process.

Earnings losses in recessions

Besides excess skewness in recessions, a key feature of the earning dynamics is its history dependence. For example, as documented in Guvenen et al. (2014), low income workers lose a significantly higher percentage of their earnings in recessions than high income individuals. We show in this Section that the endogenous earnings dynamics generated by our model is very much consistent with the above patterns of the data.

Using moments reported in Guvenen et al. (2014), we plot the cumulative earning losses as a function of their five-year cumulative earnings prior to the 2008-2010 recession in Figure 3. As in Guvenen et al. (2014), we measure individual's wealth by their five-year cumulative earnings prior to the recession and report the average drop of earnings in the by simulating a sequence of 12 quarters of $g_t = g_L$ for each individuals in each group. Figure 4 is the a counterpart of Figure 3 in our model, where we plot the median change in wage earnings in a three-year recession for workers of twenty groups ranked by their pre-recession earnings level. The vertical axis is the average change in wages for agent in the corresponding percentile. As before the dashed line describes the implication of a model with full commitment, and the dotted line is the implication of our model with limited commitment. We see that the model replicates these patterns quite well. We highlight that this particular pattern of earnings losses is entirely endogenous. Agents who received low productivity in the past are more likely to be constrained in the future and face larger average drops in labor compensation.

Asset Pricing

With a risk aversion, $\gamma = 4$ and IES $\psi = 2$ our model generates a high equity premium of 4% per year, a low risk-free interest rate of 1.3%. As remarked before, we achieve a high level of equity premium for two reasons. First, under the optimal contract, the consumption share of unconstrained agents is procyclical and more persistent than the underlying shocks. This slow moving risk factor exacerbates the volatility of the stochastic discount factor under recursive preferences. Second, our model endogenously generates a form of operating leverage because the optimal labor compensation contract must insure workers against aggregate shocks. To illustrate the quantitative importance of each channel, in Table 3, we report the key asset pricing moments in our model. The equity premium

on the aggregate consumption claim in our model is 3.4% per year, compared to 0.5% in an otherwise standard representative agent model. This additional equity premium in our model comes entirely from a more volatile stochastic discount factor. In fact, the volatility of the stochastic discount factor in our model measured as the Sharpe ratio bound is 0.5 in recessions and 0.3 in booms, almost three times of that in an otherwise identical representative agent model. Because corporate cash flow in our model has a higher exposure to aggregate shocks than aggregate consumption, the risk premium on dividend payout in our model is even higher, 4% per year. The standard deviation of excess returns on the dividend payout is about 10% or five times that in the full commitment case.

The key mechanism of our model, the fact that a countercyclical and persistent movements in labor share accounts generates a form of operating leverage for firms and affect stock market valuations is consistent with empirical evidence on the dynamics of labor share and stock market returns. The volatility of labor share in our calibrated model is about 7% per year, about the same magnitude as its counterpart in the post war U.S. data. In addition, the fact that labor share comoves with stock market valuation in our model is also consistent with Greenwald et al. (2014) who find that movements in factor shares account for a substantial movements in price-dividend ratios over longer horizons.

To illustrate the mechanism of operating leverage, we plot the expected return of firms as a function of the key state variable, u for booms (top panel) and recessions (bottom panel) in Figure 5. As discussed in the Proposition 3, high u firms are risky earn a premium of about about 6%. Thus about third of the risk premium for constrained firms comes from their higher cash-flow exposure.

Endogenous market segmentation

In our model the capital owners and the unconstrained workers share the same stochastic discount factor while the unconstrained agents' Euler equation holds with an inequality. This implication of our model can be interpreted as limited stock market participation. In fact, the procyclicality of capital owners' consumption share implies that in equilibrium, capital owners must borrow from the workers to investment in a stock that delivers aggregate consumption as dividends. In general, all agents' equilibrium consumption under the optimal contract be replicated by an appropriate portfolio strategy. The sensitivity of the value of this portfolio with respect to aggregate shock market value can be interpreted as the number of shares held by the agent in equilibrium.

To formalize the above interpretation of our calibrated model, we define $P(y, U | \phi, g)$ as the present value of a workers' compensation contract using the following recursion

$$P(y, U | \phi, g) = C(y, U | \phi, g) + (1 - \kappa) \int \Lambda(g', \eta' | \phi, g) P(ye^{g'+\varepsilon'}, U'(z') | \phi', g') \Omega(dz' | g).$$

We use homogeneity to define

$$p(u | \phi, g) = \frac{P(y, U | \phi, g)}{y},$$

and calculate Δ_C as the sensitivity of an worker's consumption portfolio with respect to the aggregate shock g :

$$\Delta_C(u, g, x) = \frac{\mathbb{E}[p(u'(u, g_H, \varepsilon' | g, x) | g_H, x'(g_H)) | u, g, x]}{\mathbb{E}[p(u'(u, g_L, \varepsilon' | g, x) | g_L, x'(g_L)) | u, g, x]}. \quad (27)$$

Intuitively, $\Delta_C(u, g, x)$ calculates the ratio of the value of a worker's compensation portfolio in booms relative to that in recessions and is a measure of its exposure to aggregate risks. A replicating portfolio with only risk free bond will have $\Delta_c = 1$ and higher values indicate higher exposure to stocks. We plot $\Delta_C(u, g, x)$ for booms and recessions (bottom panel) in Figure 6. We see that as a function of u , Δ_C is downward sloping confirming the interpretation that workers who have previously faced low productivity shocks and consequently have a current high u also have a lower exposure to equity markets.

7 Conclusion

We present an asset pricing model with endogenously incomplete market. We show that limited commitment on the principal side implies that idiosyncratic tail risks in labor income can only be partially insured. When combined with recursive preferences and a counter-cyclical tail risk in labor income, our model provides a potential resolution of the equity premium puzzle based on limited commitment of financial contracts. We also show that our model is consistent with the empirical evidence on counter-cyclical labor share and limited stock market participation.

8 Appendix

8.1 Proof of Proposition 1

Let $\mathbb{E} = \{\varepsilon' \text{ s.t the limited liability constraint equation (8) is slack}\}$. Using the first order condition in equation (16), for all $\varepsilon' \in \mathbb{E}$ define an implicit function $u'(u, \varepsilon', g' | \phi, g)$ such that $z = u'(u, \varepsilon', g' | \phi, g)$ solves

$$\left[\frac{e^{\varepsilon'} c(z | \phi', g')}{c(u | \phi, g)} \right]^{-\frac{1}{\psi}} \left[\frac{e^{\varepsilon'} z}{m(u | \phi, g)} \right]^{\frac{1}{\psi} - \gamma} = \left(\frac{x(\phi', g')}{x(\phi, g)} \right)^{-\frac{1}{\psi}} \left(\frac{w(\phi', g')}{n(\phi, g)} \right)^{\frac{1}{\psi} - \gamma}, \quad (28)$$

where $\phi' = \Gamma(g' | \phi, g)$. The Envelope condition in equation (14) and concavity of $p(u | \phi, g)$ with respect to u implies $\frac{\partial c(\cdot | \phi, g)}{\partial u} \geq 0$. Together with equation (28) this yields $\frac{\partial u'(u, \varepsilon', g' | \phi, g)}{\partial \varepsilon'} \leq 0$.

Substituting the optimal policy for u' , note that the left hand side of equation (28) is monotonic and continuous in ε' . Both $c(u | \phi, g)$ and $u'(u, g', \varepsilon' | \phi, g)$ are bounded and hence the left hand side approaches zero as $\varepsilon' \rightarrow \infty$ and approaches ∞ as $\varepsilon' \rightarrow -\infty$. For a strictly positive $x(\phi, g)$, by the Intermediate Value Theorem, there exists $\underline{\varepsilon}(u, g' | \phi, g)$ such that

$$\left[\frac{e^{\underline{\varepsilon}} c(\bar{u}(\phi', g') | \phi', g')}{c(u | \phi, g)} \right]^{-\frac{1}{\psi}} \left[\frac{e^{\underline{\varepsilon}} \bar{u}(\phi', g')}{m(u | \phi, g)} \right]^{\frac{1}{\psi} - \gamma} = \left(\frac{x(\phi', g')}{x(\phi, g)} \right)^{-\frac{1}{\psi}} \left(\frac{w(\phi', g')}{n(\phi, g)} \right)^{\frac{1}{\psi} - \gamma},$$

Monotonicity of u' with respect to ε' implies that $\mathbb{E} = [\underline{\varepsilon}(u | g' | \phi, g), \infty)$. For $\varepsilon \notin \mathbb{E}$, the limited liability constraint binds and monotonicity of p with respect to u implies that $u'(u, g', \varepsilon' | \phi, g) = \bar{u}(\phi', g')$. The right hand side of equation (28) is independent of u but the equation needs to be satisfied for all u . As before, monotonicity of $c(u | \phi, g)$ and $m(u | \phi, g)$ with respect to u is sufficient to infer that for a fixed ε' the function $u'(u, g', \varepsilon' | \phi, g)$ is increasing in the argument u too.

As $\underline{\varepsilon}(u | g' | \phi, g)$ satisfies $u'(u, g', \underline{\varepsilon}(u | g' | \phi, g) | \phi, g) = \bar{u}(g', \Gamma(g' | \phi, g))$, we have

$$\frac{\partial u'(u, \underline{\varepsilon}(u | g' | \phi, g), g' | \phi, g)}{\partial u} + \frac{\partial u'(u, \underline{\varepsilon}(u | g' | \phi, g), \varepsilon' | \phi, g)}{\partial \varepsilon'} \frac{\partial \underline{\varepsilon}(u | g' | \phi, g)}{\partial u} = 0.$$

Since $\frac{\partial u'(u, \underline{\varepsilon}(u | g' | \phi, g), g' | \phi, g)}{\partial u} \geq 0$ and $\frac{\partial u'(u, \underline{\varepsilon}(u | g' | \phi, g), \varepsilon' | \phi, g)}{\partial \varepsilon'} \leq 0$, it follows that $\frac{\partial \underline{\varepsilon}(u | g' | \phi, g)}{\partial u} \geq 0$.

8.2 Proof for Proposition 2

We assume that in period $t = 0$ all agents begin with same initial promised utility \bar{u}_0 . All policy rules depend on \bar{u}_0 , however, to keep the notation compact we drop u_0 from the list of arguments. Let $\{x_H, x_L\}$ and $\{w_H, w_L\}$ be the principal share of aggregate endowment and continuation values normalized by aggregate endowment for states $g_1 = g_H$ and $g_1 = g_L$ in period $t = 1$. We use $\{c_H, c_L(\varepsilon)\}$ and $\{u_H, u_L(\varepsilon)\}$ to denote the agent's normalized compensation and utilities in states $g_1 = g_H$ and $g_1 = g_L$ in period $t = 1$. Note that since the distribution of idiosyncratic risk is non trivial in $g_1 = g_L$, both compensation and continuation utilities depend on the realization of the idiosyncratic shocks ε . Following equation (17) the normalized utility of consuming the aggregate endowment are given by

$$u_H^{FB} = (e^{g_H} u_H^{FB})^\beta \quad u_L^{FB} = (e^{g_L} u_L^{FB})^\beta,$$

We use u_L^{CD} to denote the the normalized utility of consuming y_t every period

$$u_L^{CD} = \left(\int [e^{\{\varepsilon+g_L\}} u_L^{CD}]^{1-\gamma} f(\varepsilon|g_L) d\varepsilon \right)^{\frac{\beta}{1-\gamma}} \quad (29)$$

Proposition 1 applies and we use $\underline{\varepsilon}_L$ to denote the threshold shock such that the limited commitment constraint binds in $t = 1$. From equation (28) for $\varepsilon \geq \underline{\varepsilon}_L$, we have

$$\left[\frac{c_H}{e^{\underline{\varepsilon}_L} c_L(\underline{\varepsilon}_L)} \right]^{-1} \left[\frac{u_H}{e^{\underline{\varepsilon}_L} u_L(\underline{\varepsilon}_L)} \right]^{1-\gamma} = \left[\frac{x_H}{x_L} \right]^{-1} \left[\frac{w_H}{w_L} \right]^{1-\gamma}. \quad (30)$$

We now use the promise keeping constraint to represent continuation utility as functions of consumption:

$$\begin{aligned} w_H &= x_H^{1-\beta} n_H^\beta, \quad \text{where } n_H = (1-\alpha) e^{g_H} u_H^{FB}, \\ w_L &= x_L^{1-\beta} n_L^\beta, \quad \text{where } n_L = (1-\alpha) e^{g_L} u_L^{FB}, \end{aligned} \quad (31)$$

and,

$$\begin{aligned} u_H &= c_H^{1-\beta} m_H^\beta; \quad m_H = \alpha u_H^{FB} e^{g_H} \\ u_L(\underline{\varepsilon}_L) &= c_L(\underline{\varepsilon}_L)^{1-\beta} m_L^\beta; \quad m_L = \alpha \xi u_L^{CD} e^{g_L}, \end{aligned} \quad (32)$$

The only term that needs explanation is the certainty equivalent of agents in recessions:

$$m_L = \left\{ \int_{-\infty}^{\infty} \left[e^{g_L + \varepsilon'} (\alpha u_L^{CD}) \right]^{1-\gamma} f(\varepsilon' | g_L) d\varepsilon' \right\}^{\frac{1}{1-\gamma}} = \alpha \xi u_L^{CD} e^{g_L},$$

where we denote

$$\xi = \left\{ \int_{-\infty}^{\infty} e^{(1-\gamma)\varepsilon'} f(\varepsilon' | g_L) d\varepsilon' \right\}^{\frac{1}{1-\gamma}} \in (0, 1).$$

Now we use expressions in (31) and (32) to replace the continuation utilities in (30) and simplify to get:

$$\left[\frac{c_H}{e^{\underline{\varepsilon}_L} c_L(\underline{\varepsilon}_L) \times e^{\eta(\gamma)\underline{\varepsilon}_L}} \right]^{-\Omega(\gamma)} \left[\frac{\xi u_L^{CD}}{u_L^{MP}} \right]^{-\beta(1-\gamma)} = \left[\frac{x_H}{x_L} \right]^{-\Omega(\gamma)}, \quad (33)$$

where

$$\Omega(\gamma) = 1 + (1 - \beta)(\gamma - 1) > 0; \quad \eta(\gamma) = \frac{\beta(\gamma - 1)}{\Omega(\gamma)}; \quad 1 + \eta(\gamma) = \frac{\gamma}{\Omega(\gamma)}.$$

Market Clearing/Aggregation

The total amount of workers consumption in $g_1 = g_L$ is given by:

$$\int_{-\infty}^{\underline{\varepsilon}} e^{\varepsilon'} c_L(\underline{\varepsilon}_L) f(\varepsilon' | g_L) d\varepsilon' + \int_{\underline{\varepsilon}}^{\infty} e^{\varepsilon'} c_L(\varepsilon') f(\varepsilon' | g_L) d\varepsilon'. \quad (34)$$

The first order condition (16) implies that for all $\varepsilon \geq \underline{\varepsilon}_L$,

$$e^{-\gamma\varepsilon} c_L(\varepsilon)^{-1} u_L(\varepsilon)^{1-\gamma} = e^{-\gamma\underline{\varepsilon}_L} c_L(\underline{\varepsilon}_L)^{-1} u_L(\underline{\varepsilon}_L)^{1-\gamma}. \quad (35)$$

The promise keeping at g_L implies that for all ε ,

$$u_L(\varepsilon) = c_L(\varepsilon)^{1-\beta} m_L^\beta. \quad (36)$$

We combine equations (35) and (36) to get:

$$e^{-\gamma\varepsilon} c_L(\varepsilon)^{-1+(1-\gamma)(1-\beta)} = e^{-\gamma\underline{\varepsilon}_L} c_L(\underline{\varepsilon}_L)^{-1+(1-\gamma)(1-\beta)}.$$

Using the definition of $\Omega(\gamma)$ and $\eta(\gamma)$, we can write the above equation as

$$e^\varepsilon c_L(\varepsilon) = e^{\eta(\gamma)\varepsilon + \frac{\gamma}{\Omega(\gamma)}\underline{\varepsilon}_L} c_L(\underline{\varepsilon}_L),$$

and this holds for all $\varepsilon \geq \underline{\varepsilon}_L$. Now, we compute the integral in (34) as:

$$\begin{aligned}
& c_L(\underline{\varepsilon}_L) \int_{-\infty}^{\underline{\varepsilon}_L} e^{\varepsilon'} f(\varepsilon' | g_L) d\varepsilon' + e^{\frac{\gamma}{\Omega(\gamma)} \underline{\varepsilon}_L} c_L(\underline{\varepsilon}_L) \int_{\underline{\varepsilon}_L}^{\infty} e^{\eta(\gamma)\varepsilon} f(\varepsilon' | g_L) d\varepsilon' \\
&= \frac{\lambda}{1+\lambda} e^{-\lambda \varepsilon^{max} + (1+\lambda)\underline{\varepsilon}_L} c_L(\underline{\varepsilon}_L) + e^{\frac{\gamma}{\Omega(\gamma)} \underline{\varepsilon}_L} c_L(\underline{\varepsilon}_L) \frac{\lambda}{\lambda - \eta(\gamma)} \left[e^{-\eta(\gamma)\varepsilon^{max}} - e^{-\lambda \varepsilon^{max} + (\lambda - \eta(\gamma))\underline{\varepsilon}_L} \right] \\
&= e^{[1+\eta(\gamma)]\underline{\varepsilon}_L} c_L(\underline{\varepsilon}_L) \times \left\{ \left[\frac{\lambda}{1+\lambda} - \frac{\lambda}{\lambda - \eta(\gamma)} \right] e^{-\lambda \varepsilon^{max} + (\lambda - \eta(\gamma))\underline{\varepsilon}_L} + \frac{\lambda}{\lambda - \eta(\gamma)} e^{-\eta(\gamma)\varepsilon^{max}} \right\} \\
&= e^{[1+\eta(\gamma)]\underline{\varepsilon}_L} c_L(\underline{\varepsilon}_L) \times \left\{ \frac{\lambda}{\lambda - \eta(\gamma)} e^{-\eta(\gamma)\varepsilon^{max}} - \left[\frac{\lambda}{\lambda - \eta(\gamma)} - \frac{\lambda}{1+\lambda} \right] e^{-\lambda \varepsilon^{max} + (\lambda - \eta(\gamma))\underline{\varepsilon}_L} \right\} \tag{37}
\end{aligned}$$

For $\eta(\gamma) < \lambda$ define

$$\omega(\varepsilon) \equiv \frac{\lambda}{\lambda - \eta(\gamma)} e^{-\eta(\gamma)\varepsilon^{max}} - \left[\frac{\lambda}{\lambda - \eta(\gamma)} - \frac{\lambda}{1+\lambda} \right] e^{-\lambda \varepsilon^{max} + (\lambda - \eta(\gamma))\varepsilon}, \tag{38}$$

It is easy to verify that $\omega(\varepsilon)$ is strictly decreasing in ε , and $\omega(-\infty) = \frac{\lambda}{\lambda - \eta(\gamma)} e^{-\eta(\gamma)\varepsilon^{max}} = \frac{1+\lambda}{\lambda - \eta(\gamma)} \left[\frac{\lambda}{1+\lambda} \right]^{1+\eta(\gamma)}$, $\omega(\varepsilon^{max}) = \frac{\lambda}{1+\lambda} e^{-\eta(\gamma)\varepsilon^{max}} = \left[\frac{\lambda}{1+\lambda} \right]^{1+\eta(\gamma)}$.

General Equilibrium

In general equilibrium, we have $c_H = 1 - x_H$, and we can write (33) as:

$$\left[\frac{1 - x_L}{e^{\underline{\varepsilon}} c_L(\underline{\varepsilon}_L) \times e^{\eta(\gamma)\underline{\varepsilon}_L}} \right]^{\Omega(\gamma)} \left[\frac{\xi u_L^{CD}}{u_L^{MP}} \right]^{\beta(1-\gamma)} = \left[\frac{x_H}{x_L} \frac{1 - x_L}{1 - x_H} \right]^{\Omega(\gamma)}. \tag{39}$$

Also, from the market clearing condition, (15),

$$e^{[1+\eta(\gamma)]\underline{\varepsilon}_L} c_L(\underline{\varepsilon}_L) \omega(\underline{\varepsilon}_L) = 1 - x_L. \tag{40}$$

We show below that equations (39) and (40) are sufficient to sign $\frac{x_H}{x_L} - 1$ for any x_0 , as we $\gamma \rightarrow 1$ and $\gamma \rightarrow 1 + \lambda$. The market clearing condition (40) implies

$$\frac{1 - x_L}{e^{\underline{\varepsilon}_L} c_L(\underline{\varepsilon}_L) \times e^{\eta(\gamma)\underline{\varepsilon}_L}} = \omega(\underline{\varepsilon}_L),$$

and allows us to rewrite (39) as

$$\omega(\underline{\varepsilon}_L) \left[\frac{u_L^{FB}}{\xi u_L^{CD}} \right]^{\eta(\gamma)} = \left[\frac{x_H}{x_L} \frac{1 - x_L}{1 - x_H} \right]. \tag{41}$$

As $\gamma \rightarrow 1$, $\eta(\gamma) \rightarrow 0$ and bounds on $\omega(\cdot)$ converge to $\omega(-\infty) = 1$, $\omega(\varepsilon^{max}) = \frac{\lambda}{1+\lambda} < 1$ and from (41) $\frac{x_H}{x_L} \leq 1$.

We next show that $\lim_{\gamma \rightarrow 1+\lambda} u_L^{CD} = 0$. To see this take logs on both sides of the expression of u_L^{CD} in equation (29) to get

$$\begin{aligned} \log u_L^{CD} &= \beta \log u_L^{CD} + \beta g_H + \left(\frac{\beta}{1-\gamma} \right) \log \left[\int e^{\{(1-\gamma)\varepsilon\}} f(\varepsilon|g_L) d\varepsilon \right] \\ &= \beta \log u_L^{CD} + \beta g_H + \left(\frac{\beta}{1-\gamma} \right) \log \left(\frac{\lambda}{1-\gamma+\lambda} \right) + \beta \log \left(\frac{1+\lambda}{\lambda} \right) \end{aligned}$$

Since $\left(\frac{\lambda}{1-\gamma+\lambda} \right) \rightarrow \infty$ as $\gamma \rightarrow 1+\lambda$ and $\log u_L^{CD} \rightarrow -\infty$ which gives us $u_L^{CD} \rightarrow 0$. As

$$\frac{1-x_L}{e^{\varepsilon_L} c_L(\varepsilon_L) \times e^{\eta(\gamma)\varepsilon_L}} \geq \left[\frac{\lambda}{1+\lambda} \right]^{1+\eta(\gamma)},$$

is bounded from below, we must have $\frac{x_H}{x_L} > 1$ (in fact, $\frac{x_H}{x_L} \rightarrow \infty$).

8.3 Proof of Proposition 3

For this proof we will use $v_H(u_0)$ and $v_L(\varepsilon; u_0)$ be value of a firm in period 1 after shocks $g_1 = g_H$ and $g_1 = g_L$ for an arbitrary initial promised value u_0 . Consider initial distribution that has measure zero on u_0 and measure one on \bar{u}_0 .⁶ By definition,

$$\begin{aligned} v_H(u_0) &= 1 - c_H(u_0) + \frac{\beta}{1-\beta} x_H, \\ v_L(\varepsilon; u_0) &= 1 - c_L(\varepsilon; u_0) + \frac{\beta}{1-\beta} x_L. \end{aligned}$$

By equation (37), the expected compensation in $g_1 = g_L$ is

$$\mathbb{E}[e^\varepsilon c_L(\varepsilon; u_0)] = \theta_L e^{[1+\eta(\gamma)]\varepsilon(u_0)} \omega(\varepsilon_L(u_0)), \quad (42)$$

where we denote zero-profit level of compensation,

$$\theta_L \equiv c_L(\varepsilon_L(u_0)) = 1 + \frac{\beta}{1-\beta} x_L \quad (43)$$

⁶This distinction is important as want to establish comparative statics with respect to initial promised utility keeping state prices fixed.

to emphasize the fact $c_L(\underline{\varepsilon}_L(u_0))$ is identical for all u_0 . Therefore,

$$\begin{aligned}\mathbb{E}[e^\varepsilon v_L(\varepsilon; u_0)] &= 1 - \theta_L e^{[1+\eta(\gamma)]\underline{\varepsilon}_L(u_0)} \omega(\underline{\varepsilon}_L(u_0)) + \frac{\beta}{1-\beta} x_L \\ &= \theta_L \left[1 - e^{[1+\eta(\gamma)]\underline{\varepsilon}_L(u_0)} \omega(\underline{\varepsilon}_L(u_0)) \right].\end{aligned}$$

Similarly, we denote $\theta_H = 1 + \frac{\beta}{1-\beta} x_H$.

If \bar{u}_0 is the promised utility to the agent, then $c_H(\bar{u}_0)$ and $\{c_L(\varepsilon; \bar{u}_0)\}_\varepsilon$ must satisfy the market clearing condition:

$$c_H(\bar{u}_0) = 1 - x_H; \quad \mathbb{E}[c_L(\varepsilon; \bar{u}_0)] = 1 - x_L.$$

We have:

$$\frac{v_H(\bar{u}_0)}{\mathbb{E}[e^\varepsilon v_L(\varepsilon; \bar{u}_0)]} = \frac{1 - c_H(\bar{u}_0) + \frac{\beta}{1-\beta} x_H}{1 - E[c_L(\varepsilon; \bar{u}_0)] + \frac{\beta}{1-\beta} x_L} = \frac{x_H}{x_L}$$

In general, for arbitrary u_0 , $c_H(u_0)$ and $\{c_L(\varepsilon; u_0)\}_\varepsilon$ need not satisfy the market clearing condition. However, equations (33) and (42) still have to hold. Given the equilibrium level of x_H and x_L , we can use (33) to get

$$\frac{c_H(u_0)}{e^\varepsilon c_L(\underline{\varepsilon}_L(u_0)) \times e^{\eta(\gamma)\underline{\varepsilon}_L(u_0)}} = \frac{x_H}{x_L} \times \left[\frac{\xi u_L^{CD}}{u_L^{MP}} \right]^{\eta(\gamma)}. \quad (44)$$

Therefore, $c_H(u_0)$ can be written as

$$c_H(u_0) = \Phi(\gamma) \times e^{[1+\eta(\gamma)]\underline{\varepsilon}_L(u_0)}, \quad \text{where } \Phi(\gamma) = \frac{x_H}{x_L} \times \left[\frac{\xi u_L^{CD}}{u_L^{MP}} \right]^{\eta(\gamma)} \times \theta_L. \quad (45)$$

Note that equation (41) implies

$$\left[\frac{x_H}{x_L} \right] \left[\frac{\xi u_L^{CD}}{u_L^{MP}} \right]^{\eta(\gamma)} = \frac{1 - x_H}{1 - x_L} \omega(\underline{\varepsilon}_L) \rightarrow 0 \quad \text{as } \gamma \rightarrow 1 + \lambda.$$

This means $\Phi(\gamma) \rightarrow 0$ as $\gamma \rightarrow 1 + \lambda$.

Using (44) and (45), we have:

$$\begin{aligned}\frac{v_H(u_0)}{\mathbb{E}[e^\varepsilon v_L(\varepsilon; u_0)]} &= \frac{\theta_H - \frac{x_H}{x_L} \times \Phi(\gamma) \times \theta_L \times e^{[1+\eta(\gamma)]\underline{\varepsilon}_L(u_0)}}{\theta_L \left[1 - e^{[1+\eta(\gamma)]\underline{\varepsilon}_L(u_0)} \omega(\underline{\varepsilon}_L(u_0)) \right]} \\ &= \frac{\theta_H - \Phi(\gamma) e^{[1+\eta(\gamma)]\underline{\varepsilon}_L(u_0)}}{\theta_L \left[1 - e^{[1+\eta(\gamma)]\underline{\varepsilon}_L(u_0)} \omega(\underline{\varepsilon}_L(u_0)) \right]}.\end{aligned}$$

Our question is, under what condition

$$\frac{d}{du_0} \left[\frac{v_H(u_0)}{\mathbb{E}[v_L(\varepsilon; u_0)]} \right] > 0?$$

By Proposition 1 the threshold $\underline{\varepsilon}_L(u_0)$ is an increasing function of u_0 . Therefore, it is enough to show the following:

Claim 1. For $\gamma \rightarrow 1 + \lambda$,

$$\frac{d}{d\underline{\varepsilon}} \left[\frac{\theta_H - \Phi(\gamma) e^{[1+\eta(\gamma)]\underline{\varepsilon}}}{1 - e^{[1+\eta(\gamma)]\underline{\varepsilon}} \omega(\underline{\varepsilon})} \right] > 0.$$

Proof. It is enough to show

$$\frac{-\Phi e^{(1+\eta)\varepsilon} (1+\eta) [1 - e^{(1+\eta)\varepsilon} \omega(\varepsilon)] + [\theta_H - \Phi e^{(1+\eta)\varepsilon}] e^{(1+\eta)\varepsilon} [(1+\eta)\omega(\varepsilon) + \omega'(\varepsilon)]}{[1 - e^{(1+\eta)\varepsilon} \omega(\varepsilon)]^2} > 0.$$

We focus on the numerator and simplify. Need to show

$$\begin{aligned} & \theta_H [(1+\eta)\omega(\varepsilon) + \omega'(\varepsilon)] - \Phi e^{(1+\eta)\varepsilon} \omega'(\varepsilon) - \Phi (1+\eta) \\ &= \theta_H [(1+\eta)\omega(\varepsilon) + \omega'(\varepsilon)] - \Phi [(1+\eta) + e^{(1+\eta)\varepsilon} \omega'(\varepsilon)] > 0 \end{aligned} \quad (46)$$

Using the expression of $\omega(\varepsilon)$ in (38), we can compute

$$\begin{aligned} (1+\eta)\omega(\varepsilon) + \omega'(\varepsilon) &= (1+\eta) \frac{\lambda}{\lambda-\eta} \left[e^{-\eta\varepsilon^{max}} - e^{-\lambda\varepsilon^{max} + (\lambda-\eta)\varepsilon} \right] \\ &= (1+\eta) \frac{\lambda}{\lambda-\eta} e^{-\eta\varepsilon^{max}} \left[1 - e^{-(\lambda-\eta)\varepsilon^{max} + (\lambda-\eta)\varepsilon} \right] \\ &= (1+\eta) \frac{\lambda}{1+\lambda} e^{-\eta\varepsilon^{max}} \frac{1+\lambda}{\lambda-\eta} \left[1 - e^{-(\lambda-\eta)\varepsilon^{max} + (\lambda-\eta)\varepsilon} \right] \\ &= (1+\eta) e^{-(1+\eta)\varepsilon^{max}} \frac{1+\lambda}{\lambda-\eta} \left[1 - e^{-(\lambda-\eta)(\varepsilon^{max}-\varepsilon)} \right], \end{aligned}$$

where the last line uses the fact $\varepsilon^{max} = \ln \frac{1+\lambda}{\lambda}$. Also, the second in equation (46) can be written as

$$\begin{aligned} (1+\eta) + e^{(1+\eta)\varepsilon} \phi'(\varepsilon) &= (1+\eta) \left[1 - \frac{\lambda}{1+\lambda} e^{-\lambda\varepsilon^{max} + (1+\lambda)\varepsilon} \right] \\ &= (1+\eta) \left[1 - e^{-(1+\lambda)(\varepsilon^{max}-\varepsilon)} \right]. \end{aligned}$$

Therefore, to prove (46), it is enough to show:

$$\theta_H e^{-(1+\eta)\varepsilon^{max}} \frac{1+\lambda}{\lambda-\eta} \left[1 - e^{-(\lambda-\eta)(\varepsilon^{max}-\varepsilon)} \right] - \Phi \left[1 - e^{-(1+\lambda)(\varepsilon^{max}-\varepsilon)} \right] > 0.$$

Because $\Phi(\gamma) \rightarrow 0$ as $\gamma \rightarrow 1 + \lambda$, we have $\theta_H e^{-(1+\eta)\varepsilon^{max}} > \Phi$ for γ close enough to $1 + \lambda$. We complete the proof by making the following observation:

Define

$$\begin{aligned} f(\varepsilon) &= \frac{1+\lambda}{\lambda-\eta} \left[1 - e^{-(\lambda-\eta)(\varepsilon^{max}-\varepsilon)} \right] \\ g(\varepsilon) &= 1 - e^{-(1+\lambda)(\varepsilon^{max}-\varepsilon)}, \end{aligned}$$

then $f(\varepsilon) > g(\varepsilon)$ for all $\varepsilon < \varepsilon^{max}$.

To proof the above claim, note that $f(\varepsilon^{max}) = g(\varepsilon^{max}) = 0$. It is enough to show that $f'(\varepsilon) < g'(\varepsilon)$ for all $\varepsilon < \varepsilon^{max}$, which is clearly true as

$$\begin{aligned} f'(\varepsilon) &= -(1+\lambda) e^{-(\lambda-\eta)(\varepsilon^{max}-\varepsilon)} \\ g'(\varepsilon) &= -(1+\lambda) e^{-(1+\lambda)(\varepsilon^{max}-\varepsilon)}. \end{aligned}$$

□

8.4 Proof for Lemma 1

The SDF when Y_t follows (21) is

$$\Lambda(g', \eta' | \phi, g) = \beta \left[\frac{x(\phi', g') e^{g'+\sigma(g')\eta'}}{x(\phi, g)} \right]^{-\frac{1}{\psi}} \left[\frac{w(\phi', g') e^{g'+\sigma(g')\eta'}}{n(\phi, g)} \right]^{\frac{1}{\psi}-\gamma}, \quad (47)$$

with $w(\cdot)$ and $n(\cdot)$ in equations (12) and (13) appropriately modified.

The optimality conditions (16) now are modified such that all possible realizations of $(\varepsilon', g', \eta')$,

$$\beta \left[\frac{e^{g'+\sigma(g')\eta'+\varepsilon'} c(u'(\varepsilon', g', \eta' | \phi, g) | \phi', g')}{c(u | \phi, g)} \right]^{-\frac{1}{\psi}} \left[\frac{e^{g'+\sigma(g')\eta'+\varepsilon'} u'(\varepsilon', g', \eta' | \phi, g)}{m(u | \phi, g)} \right]^{\frac{1}{\psi}-\gamma} \geq \Lambda(g', \eta' | \phi, g), \quad (48)$$

and $u'(\varepsilon', g', \eta' | \phi, g) < \bar{u}(\phi', g')$ implies that "="" must hold in (48), where $m(u | \phi, g)$ is the

modified certainty equivalent defined in (7).

When $u'(\varepsilon', g', \eta' | \phi, g) < \bar{u}(\phi', g')$, (48) simplifies to

$$\left[\frac{e^{\varepsilon'} c(u'(\varepsilon', g', \eta' | \phi, g) | \phi', g')}{c(u | \phi, g)} \right]^{-\frac{1}{\psi}} \left[\frac{e^{\varepsilon'} u'(\varepsilon', g', \eta' | \phi, g)}{m(u | \phi, g)} \right]^{\frac{1}{\psi} - \gamma} = \left[\frac{x(\phi', g')}{x(\phi, g)} \right]^{-\frac{1}{\psi}} \left[\frac{w(\phi', g')}{n(\phi, g)} \right]^{\frac{1}{\psi} - \gamma} \quad (49)$$

From (49), we can get $u'(g', \eta'_1, \varepsilon' | \phi, g) = u'(g', \eta'_2, \varepsilon' | \phi, g)$ for any η'_1 and η'_2 since c is a non-decreasing function in u' . It means the first order condition does not depend on η' when $u'(\varepsilon', g', \eta' | \phi, g) < \bar{u}(\phi', g')$.

When $u'(\varepsilon', g', \eta' | \phi, g) = \bar{u}(\phi', g')$, we can get that $u'(\varepsilon', g', \eta' | \phi, g) = \bar{u}(\phi', g')$ for any η' . Since $\bar{u}(\phi', g')$ does not depend on η' , the first order condition does not depend on η' when $u'(\varepsilon', g', \eta' | \phi, g) = \bar{u}(\phi', g')$. Since we characterize the normalized optimal contracting problem by some equivalent first order conditions which do not depend on η and η' , our assumption that z does not depend on η is correct. We can always find an equilibrium such that η is not a state variable.

A Details of the negative exponential distribution

We assume that the density function of the idiosyncratic shock $f_{g_L}(\varepsilon)$ takes the following form:

$$f(\varepsilon) = \begin{cases} 0 & \varepsilon > \varepsilon^{max} \\ \lambda e^{\lambda(\varepsilon - \varepsilon^{max})} & \varepsilon \leq \varepsilon^{max} \end{cases}$$

The integral of $f(\varepsilon)$ is

$$\int_{-\infty}^y f(\varepsilon) d\varepsilon = e^{\lambda(y - \varepsilon^{max})}.$$

In particular, $\int_{-\infty}^{\infty} f(\varepsilon) d\varepsilon = 1$ is a proper density. Also,

$$\int_{-\infty}^y e^{\theta\varepsilon} f(\varepsilon) d\varepsilon = \frac{\lambda}{\lambda + \theta} e^{-\lambda\varepsilon^{max} + (\theta + \lambda)y} \quad \text{for } \lambda + \theta > 0.$$

In particular,

$$\int_{-\infty}^{\infty} e^{\theta\varepsilon} f(\varepsilon) d\varepsilon = \frac{\lambda}{\lambda + \theta} e^{\theta\varepsilon^{max}} \quad \text{for } \lambda + \theta > 0.$$

With $\theta = 1$, the requirement $E[e^\varepsilon] = 1$ requires that $\varepsilon^{max} = \ln \frac{1+\lambda}{\lambda}$.

B Computational Algorithm

Below we describe briefly our computation algorithm.

1. Start with an initial guess of the law of motion of x as in equation (26). Given the law of motion of x , compute the SDF $\Lambda(g', \eta' | g, x)$ using equation (22).
2. With the SDF, we solve the following optimal contracting problem

$$\begin{aligned}
p(u|g, x) &= \max_{c, \{u'(z')\}} \left\{ (1-c) + (1-\kappa) \int \Lambda(g', \eta' | g, x) e^{g' + \sigma(g')\eta' + \varepsilon'} p(u'(z') | g', x') \Omega(dz' | g) \right\} \\
s.t. \quad u &= \left[(1-\beta) c^{1-\frac{1}{\psi}} + \beta m(u|g, x)^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, \\
m(u|g, x) &= \left\{ (1-\kappa) \int \left[e^{g' + \sigma(g')\eta' + \varepsilon'} u'(z') \right]^{1-\gamma} d\Omega(z' | g) \right\}^{\frac{1}{1-\gamma}}, \\
u'(z') &\geq \underline{u}(g'), \quad \text{for all } z', \\
p(u'(z') | g', x') &\geq 0, \quad \text{for all } z'.
\end{aligned}$$

This step yields i) the value function $p(u|g, x)$ and the cutoff value of $\bar{u}(g, x)$ that satisfies $p(\bar{u}(g, x) | g, x) = 0$; and ii) the policy functions $c(u|g, x)$ and $u'(u, g', \varepsilon' | g, x)$, which specifies the law of motion of u' .

3. In simulations, we approximate the continuous distribution ϕ by a finite-state distribution as follows. We fix u_1, u_2, \dots, u_N , where $u_1 = \min_g u_{MIN}(g)$ and $u_N = \max_{g,x} \bar{u}(g, x)$. A density ϕ is described by two vectors $\{\phi[n](t), u^*[n](t)\}_{n=1}^{N+1}$ such that

- $u^*[1]$ and $u^*[N+1]$ are the boundaries where the limited commitment constraint binds: $u^*[1] = u_{MIN}(g_t)$ and $u^*[N+1] = \bar{u}(g_t, x_t)$.
- $\{u^*[j]\}_{j=2,3,\dots,N}$ are the interior points: $u^*[j] \in (u_{j-1}, u_j)$, for $j = 2, 3 \dots N$, are chosen appropriately to minimize the approximation error.
- $\phi[1]$ and $\phi[N+1]$ are the income shares of agents with a binding limited commitment constraint at $u^*[1]$ and $u^*[N+1]$, respectively.
- $\{\phi[j]\}_{j=2,3,\dots,N}$ are the income shares of agents in the interior.

Clearly, we should have $\sum_{j=1}^{N+1} \phi [j] (t) = 1$ for all t .

4. Starting with an initial distribution of u , denoted $\phi_0 (u)$. For example, we can choose a point mass at $u_0 (g)$ with $u_{IMN} (g) < u (g) < \bar{u} (g, x)$ for every x, g , solve the following equation for x_0 :

$$\int \phi_0 (u) c (u | g, x_0) du = 1 - x_0$$

Now we have an initial condition (ϕ_0, x_0) , and ϕ_0 is represented as $\{\phi [n] (0), u^* [n] (0)\}_{n=1}^{N+1}$, such that $\phi [m] (0) = 1$, and $u^* [m] (0) = u_0 (g_0)$, where m is defined by $u_0 (g) \in (u_{m-1}, u_m)$.

5. Having solved x_0 , we use the law of motion of $u' (u, g', \varepsilon' | g, x)$ to compute ϕ_1 . Here we describe a general procedure to solve for $\{\phi [n] (t+1); u^* [n] (t+1); x_{t+1}\}_{n=1}^{N+1}$ given $\{\phi [n] (t); u^* [n] (t); x_t\}_{n=1}^{N+1}$. Note that the assumed law of motion gives a natural candidate for x_{t+1} . We denote $x_{t+1} = \Gamma (x_t | g_t, g_{t+1})$.

- (a) First, we approximate the distribution $f (\varepsilon | g)$ by a finite dimensional distribution such that $\sum_j^J f_g [j] = 1$ and $\sum_j^J e^{\varepsilon_j} f_g [j] = 1$, for $g = g_H, g_L$.
- (b) Given $\{\phi [n] (t), u^* [n] (t)\}_{n=1}^{N+1}$ for period t , conditioning on the realization of aggregate state g_{t+1} , for each $n = 1, 2, \dots, N$, we compute $\{\phi_{t+1} [n, j]\}_{n,j}$. The interpretation is that $\phi_{t+1} [n, j]$ is the total measure of income share that comes from agents with $u^* [n] (t)$ and with realization of ε_j , which is given by:

$$\phi_{t+1} [n, j] = f_{g_{t+1}} [j] \phi_t [n] e^{\varepsilon_j}, \quad j = 1, 2, \dots, J.$$

Note that the continuation utility of these agents is $u' (u^* [n] (t), g_{t+1}, \varepsilon_j | g_t, x_t)$, a fact that we will use below.

- (c) We now compute $\{\phi_{t+1} [m]\}_{m=1,2,\dots,N+1}$ for the next period.

$$\begin{aligned} \phi_{t+1} [m] &= \sum_{n=1}^{N+1} \sum_{j=1}^J \phi_{t+1} [n, j] I_{\{u'(u^*[n](t), g_{t+1}, \varepsilon_j | g_t, x_t) \in [u_{m-1}, u_m]\}}, \quad m = 2, 3, \dots, N \\ \phi_{t+1} [1] &= \sum_{n=1}^{N+1} \sum_{j=1}^J \phi_{t+1} [n, j] I_{\{u'(u^*[n](t), g_{t+1}, \varepsilon_j | g_t, x_t) \leq u_{MIN}(g_{t+1})\}}, \\ \phi_{t+1} [N+1] &= \sum_{n=1}^N \sum_{j=1}^J \phi_{t+1} [n, j] I_{\{u'(u^*[n](t), g_{t+1}, \varepsilon_j | g_t, x_t) \geq \bar{u}(g_{t+1}, x_{t+1})\}}. \end{aligned}$$

The interpretation is again that $\phi[1]$ and $\phi[N+1]$ are the income shares of agents with a binding limited commitment constraint at $u^*[1]$ and $u^*[N+1]$, respectively, and $\{\phi[j]\}_{j=2,3,\dots,N}$ are the income shares of agents in the interior.

- (d) We need to update the vector normalized utilities $\{u^*[n](t+1)\}_{n=1}^{N+1}$. Clearly, we should have $u^*[1](t+1) = u_{MIN}(g_{t+1})$ and $u^*[N+1](t+1) = \bar{u}(g_{t+1}, x_{t+1})$. For $m = 2, 3, \dots, N$, we choose $u^*[m](t+1) \in [u_{m-1}, u_m]$ such that the resource constraint holds exactly for $u \in [u_{m-1}, u_m]$. That is, we pick $u^*[m](t+1)$ to be the solution (denoted u^*) to

$$\begin{aligned} & \sum_{n=1}^{N+1} \sum_{j=1}^J \phi_{t+1}[n, j] c(u'(u^*[n](t), g_{t+1}, \varepsilon_j | g_t, x_t) | g_{t+1}, x_{t+1}) I_{\{u'(u^*[n](t), g_{t+1}, \varepsilon_j | g_t, x_t) \in [u_{m-1}, u_m]\}} \\ = & c(u^* | g_{t+1}, x_{t+1}) \phi_{t+1}[m]. \end{aligned}$$

6. Up to now, we have described a procedure to simulate forward the economy. This allows us to compute the "market clearing" $\{x_{t+1}^{MC}\}_{t=0}^{\infty}$ as follows:

$$x_{t+1}^{MC} = 1 - \sum_{m=1}^{N+1} c(u^*[m](t+1) | g_{t+1}, x_{t+1}) \phi_{t+1}[m]. \quad (50)$$

Given the sequence of $\{g_t\}_{t=1}^T$, we simulate the economy forward for T periods to obtain $\{x_t^{MC}\}_{t=0}^T$. We divide the sample into four cases: $g_H \rightarrow g_H$, $g_H \rightarrow g_L$, $g_L \rightarrow g_H$, $g_L \rightarrow g_L$ and use regression to update the law of motion of x . We go back to step 1 to iterate.

Note that under the above procedure, given the sequence of $\{g_t\}_{t=1}^T$, the sequence of x_{t+1} that is used for computing decision rules is completely determined by (??). In the simulation, we assume that x_{t+1} follows the perceived law of motion, based on which agents make their decisions. We use the market clearing condition to update the actual law of motion of x and iterate.

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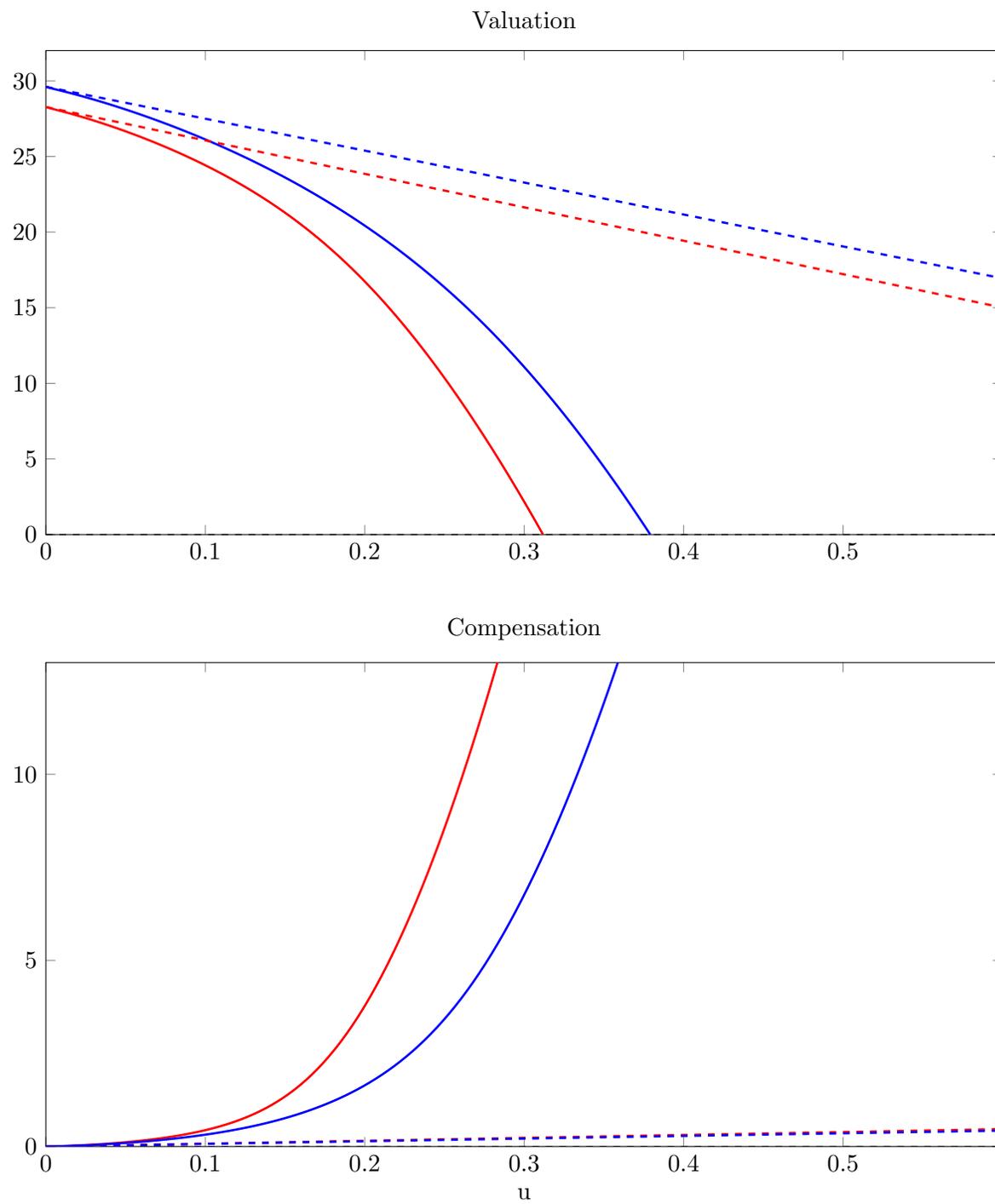


Figure 1: Firm valuations and policy rule for worker's compensation as functions of normalized promised utility

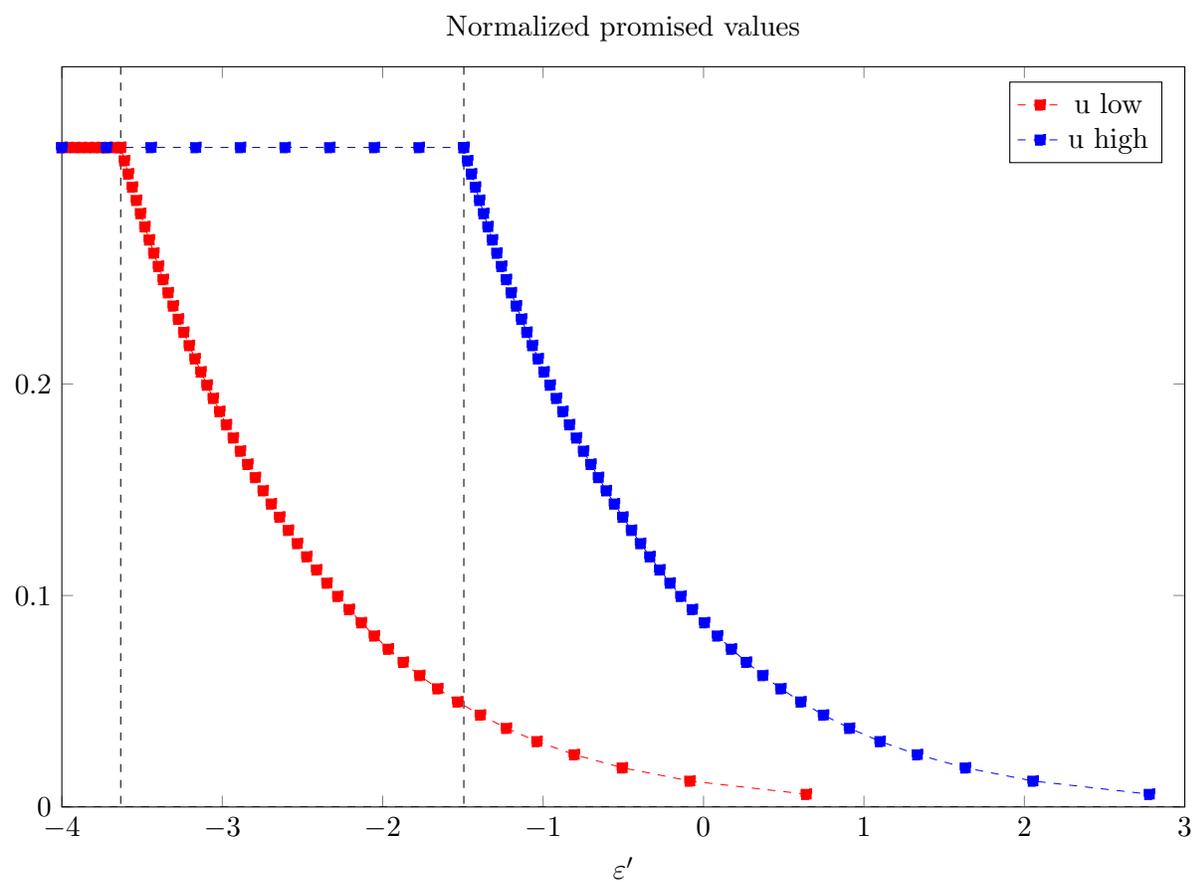


Figure 2: Normalized promised values as a function of the idiosyncratic shock ε'

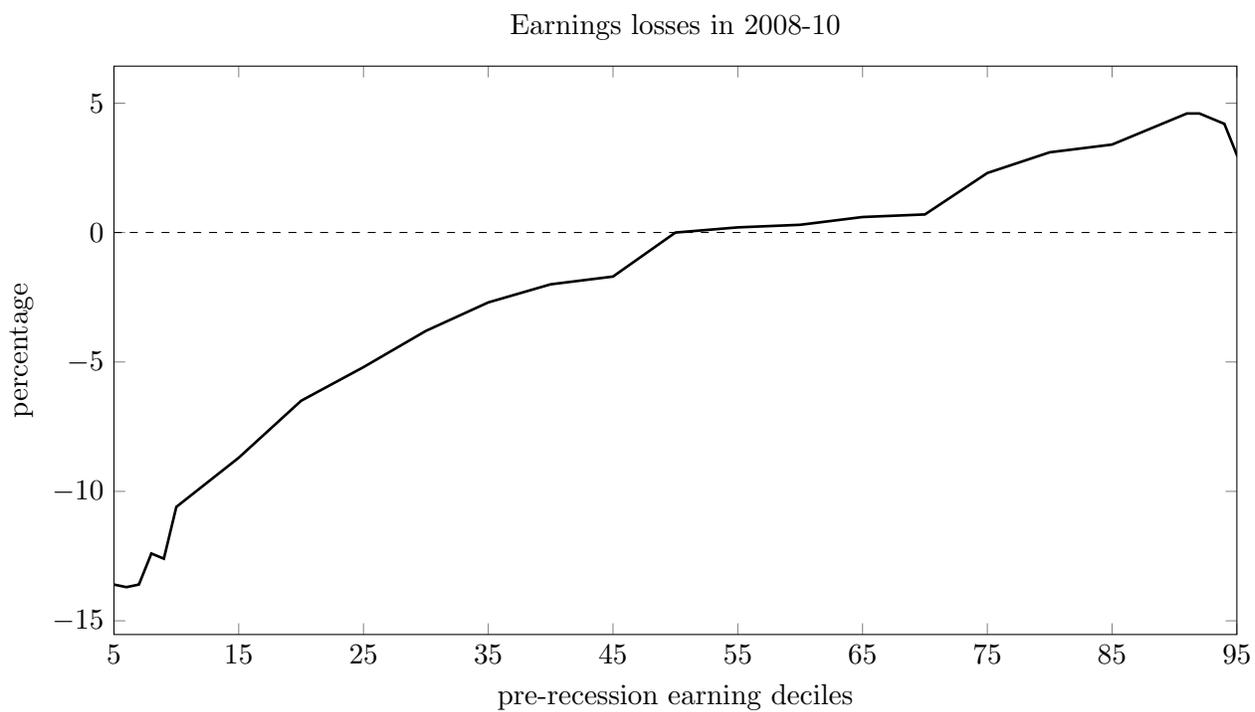


Figure 3: Demeaned earning losses in 2008-2010 recession [Data: Guvenen et al. (2014)]

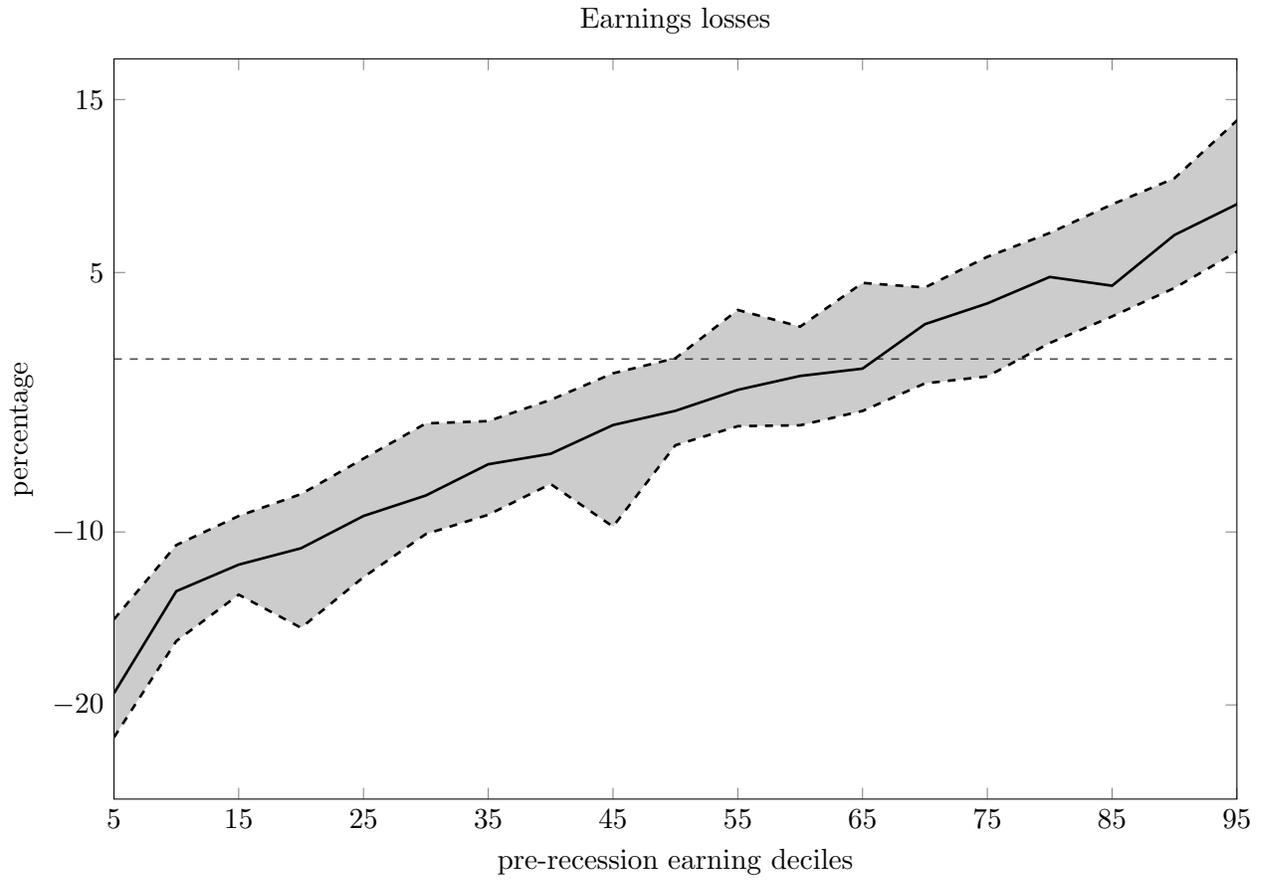


Figure 4: Median earnings losses for 12 quarters of g_L . The shaded region is the 95% confidence interval.

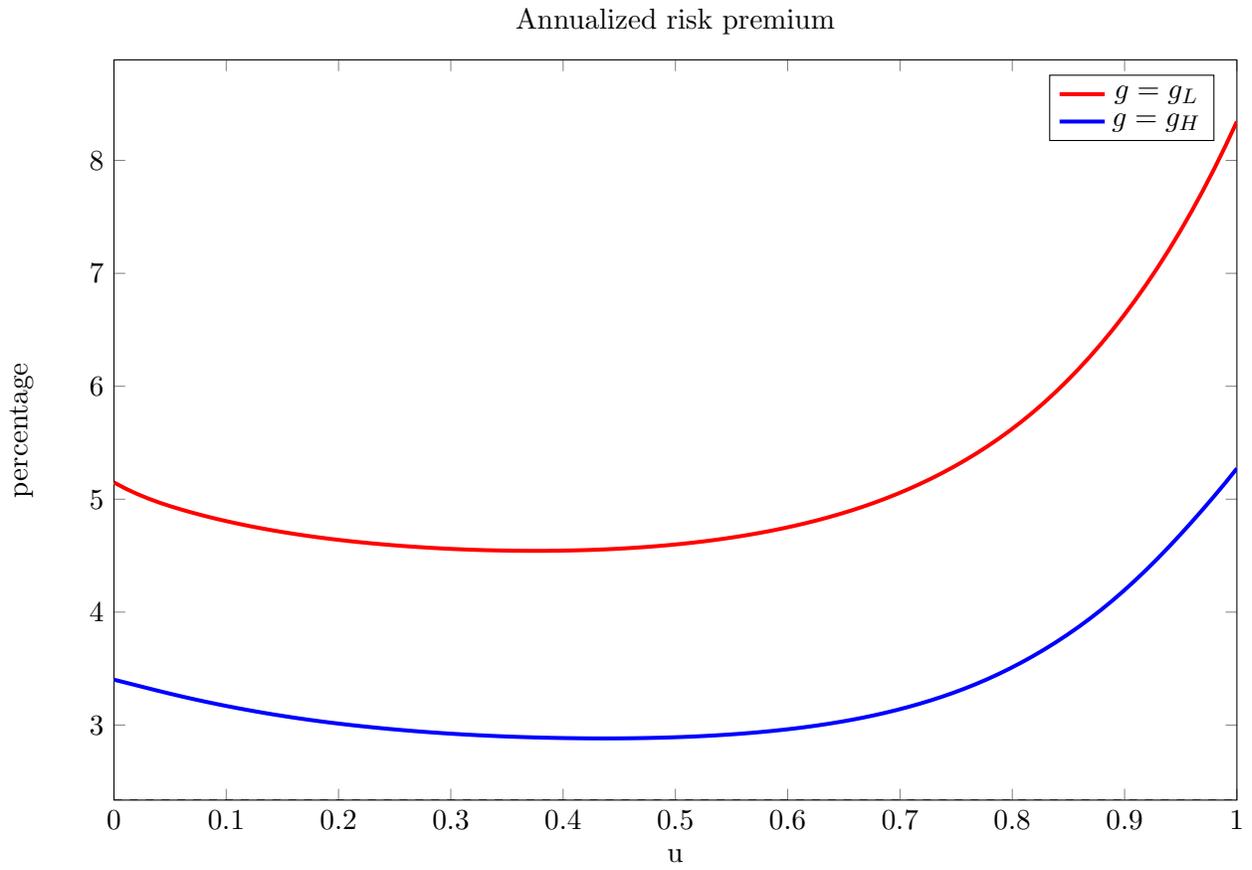


Figure 5: Annualized equity premium on firms that differ in u . The x -axis is normalized such that $\underline{u}(g) = 0$ and $\bar{u}(g) = 1$ for $g = g_L$ and $g = g_H$.

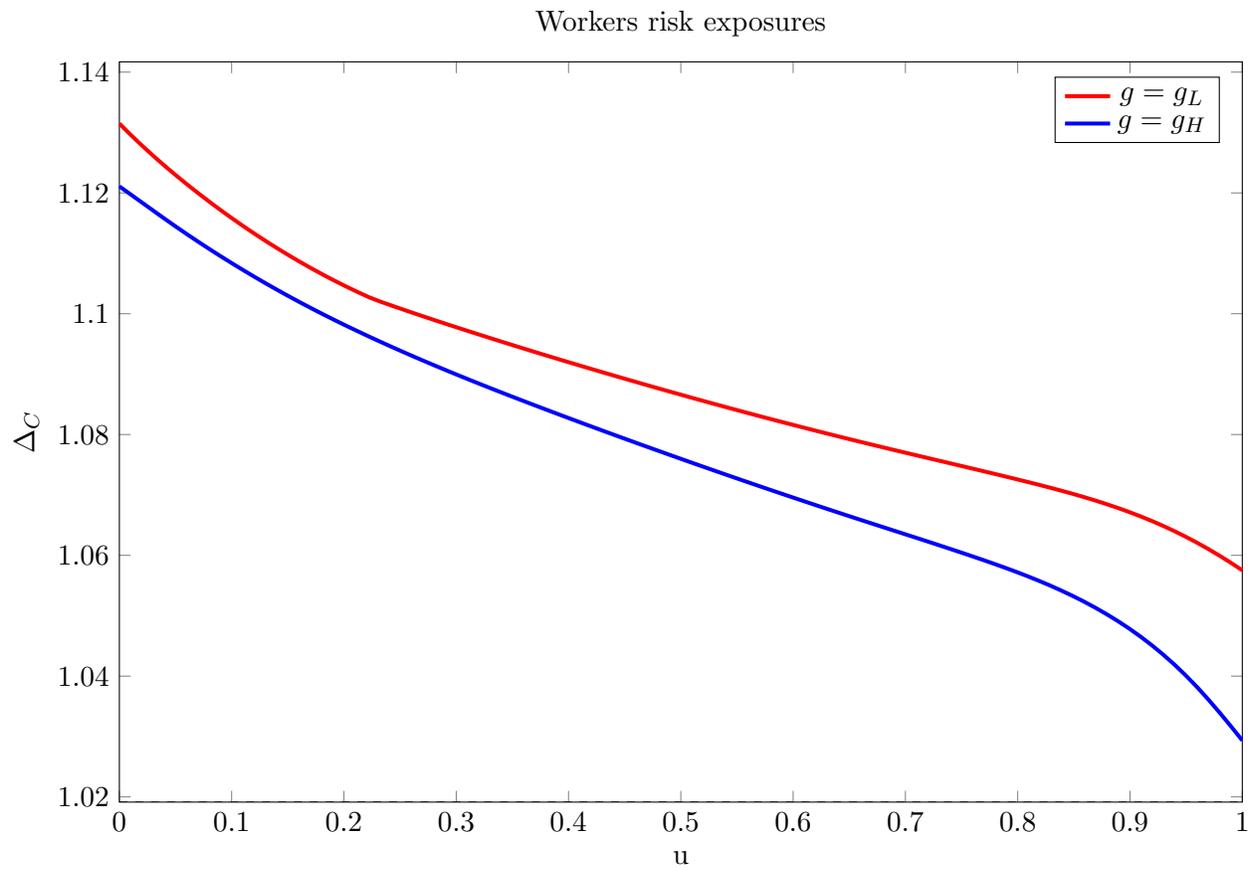


Figure 6: Workers' risk exposure measured using equation (27). The x -axis is normalized such that $\underline{u}(g) = 0$ and $\bar{u}(g) = 1$ for $g = g_L$ and $g = g_H$.

Parameters	Values
Aggregate Risk	
g_H, g_L	0.7%, -0.3%
$\sigma(g_H), \sigma(g_L)$	0.9%, 1.3%
$\pi(g_H g_H), \pi(g_L g_L)$	0.98, 0.94
Idiosyncratic Risk	
σ_H	10.3%
σ_L	10.3%
ρ_L	0.18
λ_L	4.5
$\underline{\alpha}, \bar{\alpha}$	0.16, 0.25

Table 1: Parameters for aggregate and idiosyncratic shocks

Moments	Data	Model
Std. of 1 yr earnings growth in booms	0.5	0.3
Std. of 1 yr earnings growth in recessions	0.5	0.4
Log50-Log 10 of 1yr earnings growth in booms	0.4	0.4
Log50-Log 10 of 1yr earnings growth in recessions	0.5	0.6
Kelly skewness of 1 yr earnings in growth booms	3%	-2%
Kelly skewness of 1 yr earnings growth in recessions	-9%	-6%
Average share of labor compensation in output	63%	61%

Table 2: Model fit.

Moments	Full commitment	Limited commitment
Equity premium on Y_t	0.5%	3.2%
Volatility of Equity premium on Y_t	2.2%	10.3%
Equity premium on $x_t Y_t$	0.5%	3.7%
Volatility of Equity premium on $x_t Y_t$	2.2%	12.4%
Sharpe ratio bounds $\sigma(\log \Lambda g_L), \sigma(\log \Lambda g_H)$	0.21, 0.12	0.44, 0.24
Average risk free rate	4.6%	1.6%
Volatility of risk free rate	0.4%	2.3%

Table 3: Asset pricing implications. All moments are reported at annualized frequencies.